until the yard was capable of supporting the customary load of 40 ft to 50 ft of ore (3 tons to 4 tons per sq ft). Conversely, the strength of the preconsolidated deposits appears not to have increased as a consequence of the ore loading. There is some evidence that with successive applications of load at low stress levels the deformation at a given level of stress becomes smaller.

Tierods between the main retaining walls are capable of increasing the capacity of a yard, but by a small amount. Structural platforms supported by piles or piers have been successful but only when full consideration has been given to the shearing stresses exerted on the slab by the ore pile, and to the lateral forces that may be exerted on the platform by soil-supported ore piles in adjacent areas. Sheet pile cells beneath the central part of the ore pile seem to have been beneficial but the data are insufficient to permit a reliable assessment of their value.

The theory of a perfectly cohesive material squeezed between two rough rigid plates, when adapted to the ore storage problem, provides a means for evaluating the relative stability of the yards. The results are in reasonable agreement with experience. The influence of tie rods and other structural features can be taken into account. It is found that for maximum over-all stability the surface of the ore pile should slope upward from an initial height near the dock at a rate that depends on the thickness and strength of the clay. Some existing ore yards are shown to satisfy these conditions more closely than others, and to have experienced more satisfactory behavior.

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LATERAL RESISTANCE OF PILES IN COHESIONLESS SOILS

By Bengt B. Broms,[1] M. ASCE

SYNOPSIS

Herein methods are presented for the determination of lateral deflections at working loads and the ultimate resistance of laterally loaded piles driven into cohesionless soils. The lateral deflections have been computed assuming that the coefficient of subgrade reaction increases linearly with depth. The evaluation of the coefficient of subgrade reactions is discussed and the calculated lateral deflections are compared with available test data.

The ultimate lateral resistance of laterally loaded piles has been assumed to be governed by the lateral resistance of the surrounding cohesionless soil or by the yield or ultimate moment resistance of the pile sections. It has been assumed further that the ultimate lateral resistance of a cohesionless soil is equal to three times the Rankine passive pressure. The calculated ultimate lateral resistances are compared with available test data. Satisfactory agreement is found between measured and calculated values.

INTRODUCTION

This paper is the second of two papers concerned with the lateral resistance of vertical single piles and of pile groups. In the first paper, the lateral

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[1] Assoc. Prof. of Civ. Engrg., Cornell Univ., Ithaca, N. Y.

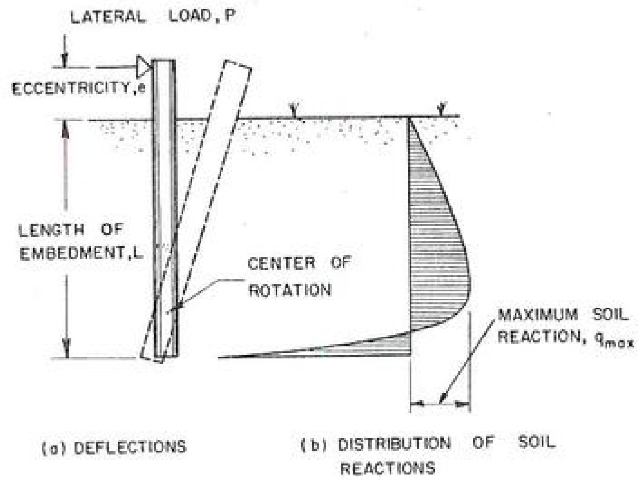

FIG. 1.—FAILURE MODE OF A SHORT FREE-HEADED PILE

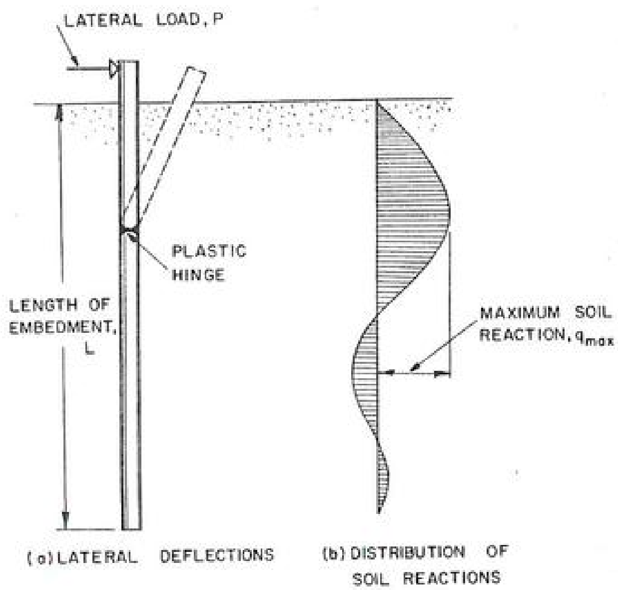

FIG. 2.—FAILURE MODE OF A LONG FREE-HEADED PILE

resistance of piles driven into cohesive soils (clays) has been investigated. In this paper, the lateral resistance of piles driven into cohesionless soils (sands) is investigated.

The design of a laterally loaded pile group is, in general, governed by the deflections at working loads and by the ultimate strength of the pile group. Methods for the calculation of lateral deflections will be presented subsequently herein.

Failure of a laterally loaded pile takes place either when the maximum bending moment in the loaded pile reaches the ultimate or yield resistance of the pile section or when the lateral earth pressures reach the ultimate lateral resistance of the soil along the total length of the pile.

The lateral deflections and the distribution of soil reactions or lateral earth pressures at failure for a relatively short unrestrained pile are illustrated in Fig. 1. Failure takes place when the pile rotates as a unit with respect to a point located close to its toe. The lateral earth pressures increase with depth and reach a maximum at some depth below the ground surface close to the center of rotation. The negative lateral earth pressures which develop at the toe or tip of the laterally loaded pile are large.

The mode of failure of a long pile is shown in Fig. 2. Failure takes place when the maximum bending moment exceeds the yield resistance of the pile section and a plastic hinge forms at the section of maximum bending moment. The corresponding lateral deflections and distribution of soil reactions are shown also in Fig. 2. The lateral deflections of the pile above the plastic hinge will be large. It will be assumed in the following analysis that the full passive lateral resistance develops at failure in the surrounding soil. The lateral deflections of the pile located below the location of the plastic hinge will be relatively small and the ultimate lateral passive resistance of the soil may not be fully developed along this part of the pile.

Methods for the calculation of the ultimate lateral resistance of piles driven into cohesionless soils are developed herein.

The lateral deflections and the distribution of bending moments for piles driven into cohesionless soils have been measured by many investigators.[2 to 38]

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16 Matuo, H. and Takahasi, T., "The Lateral Resistance of Piles," Report No. 42, Research Inst. of Civil Engineering, Ministry of Home Affairs, Japan, Aug. 1938, 9 pp.

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In general, the field or laboratory tests have not been carried out to failure and the ultimate lateral resistances of the test piles are usually not known. Frequently, sufficient information with respect to the strength properties, unit weight of the supporting soil or the location of the ground water table are not available to permit evaluation of the test data. Therefore, a comparison between measured and calculated ultimate lateral resistance is frequently difficult or impossible to make. When sufficient information is available, the calculated ultimate lateral resistances have been compared with the test data.

Notation.—Letter symbols adopted for use in this paper are defined where they first appear and listed alphabetically in Appendix I.

LATERAL DEFLECTIONS AT WORKING LOADS

Coefficient of Subgrade Reaction.—Terzaghi[39] has shown that the coefficient of lateral subgrade reaction k_h (in pounds per cubic inch or tons per cubic foot) can, for a cohesionless soil, be assumed to increase approximately linearly with depth and to decrease linearly with increasing width D of the loaded area as expressed by the equation:

$$k_h = n_h \frac{z}{D} \qquad (1)$$

in which n_h (in pounds per cubic inch or tons per cubic foot) is the coefficient of horizontal subgrade reaction for a long strip with a width of unity at a depth of unity below the ground surface. Thus it is assumed that the coefficient of subgrade reaction is independent of the stiffness of the pile and of the pile length. Approximate values of the coefficient n_h are given in Table 1. These values are the same as those recommended by Terzaghi.[39] The coefficient of subgrade reaction n_h is assumed to vary only with the relative density of the soil and with the location of the ground water table.

The largest part of the total deflections of a laterally loaded pile driven into a cohesionless soil takes place at the time of loading. Only a small increase of the deflections takes place subsequent to loading. This increase is caused by creep through rearrangement of the individual soil particles. However, it should be noted that repetitive loading and vibration may cause a substantial increase of the deflections, especially if the relative density of the surrounding soil is low. The limited amount of available test data indicates

33 Vierhelier, H. A., "Lateral Load Tests Made on Steel Bearing Piles," Engineering News Record, Vol. 118, May 6, 1937, pp. 667-669.

34 Wagner, A. A., "Lateral Load Tests on Piles for Design Information," Symposium on Lateral Load Tests on Piles, ASTM Special Publication, No. 154, 1953, pp. 59-72.

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36 Wen, R. K. L., "Model Studies of Laterally Loaded Pile Foundations," Proceedings, 34th Annual Meeting of the Highway Research Board, Vol. 34, 1955, pp. 140-152.

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38 Williams, T. E. H., "Soil Failure Due to Overturning of Piles," Engineering, Vol. 173, No. 4488, London, February, 1952, pp. 134-136.

39 Terzaghi, K., "Evaluation of Coefficients of Subgrade Reaction," Géotechnique, Institution of Civ. Engrs., Vol. V, London, 1955, pp. 297-326.

TABLE 1.—EVALUATION OF COEFFICIENT n_h TERZAGHI[39], 1955

Relative Density	Coefficient n_h, in tons per cubic feet		
	Loose	Medium	Dense
Above ground water table	7	21	56
Below ground water table	4	14	34

that the effects of repetitive loading can be taken into account through a reduction of the coefficient of lateral subgrade reaction.

Lateral Deflections.—Rifaat[40] and others[41,42,43] have suggested that the lateral deflections at the ground surface y_o of a single pile or of a pile group can be expressed at working loads in terms of the dimensionless deflection $y_o\ (EI)^{3/5}\ (n_h)^{2/5}/PL$ and $y_o\ (EI)^{4/5}\ (n_h)^{1/5}/ML$ and the dimensionless depth of embedment η L in which η is equal to $(n_h/EI)^{1/5}$. The quantity EI is the stiffness of the pile section, P and M the applied lateral load and moment at the ground surface, respectively, and L the embedment length (Figs. 1 and 2). The dimensionless lateral deflection at the ground surface has been plotted in Fig. 3 as a function of the dimensionless embedment length η L.

It can be seen from Fig. 3 that a laterally loaded pile behaves as an infinitely stiff member when the dimensionless length η L is less than about 2.0 and as an infinitely long member when the dimensionless length η L exceeds about 4.0. For short piles (η L less than 2.0), an increase of the embedment length decreases the lateral deflections at the ground surface, while for long piles (η L larger than 4.0), the lateral deflections at the ground surface are unaffected by a change of the embedment length. On the other hand, an increase in the stiffness EI of the pile section decreases the lateral deflections of long piles but does not affect the lateral deflections of short piles.

Long Piles.—The lateral deflection y_o at the ground surface can be calculated directly for a long unrestrained pile (η L larger than 4.0) from

$$y_o = \frac{2.40\ P}{n_h^{3/5}\ (EI)^{2/5}} \quad \text{(2a)}$$

and for a long (η L larger than 4.0) fully restrained pile, the lateral deflection y_o can be calculated from

$$y_o = \frac{0.93\ P}{n_h^{3/5}\ (EI)^{2/5}} \quad \text{(2b)}$$

40 Rifaat, I. J., "Die Spunwant als Erddruck Problem," A. G. Gebr. Leeman & Co., Zürich, 1935.

41 Blum, H., "Beitrag zur Berechnung von Bohlwerken unter Berücksichtigung der Wandverformung, ins besondere bei mit der Tiefe linear zunehmender Widerstandsziffer," Wilhelm Ernest & Son, Berlin, 1951.

42 Kidder, R. E., "Lateral Loading of Vertical Foundation Piling," Project 5112, California Research Corp., August 6, 1954.

43 Reese, L. C. and Matlock, H., "Non-Dimensional Solutions for Laterally Loaded Piles with Soil Modulus Assumed Proportional to Depth," Proceedings, 8th Texas Conf. on Soil Mechanics and Foundations, 1956, 41 pp.

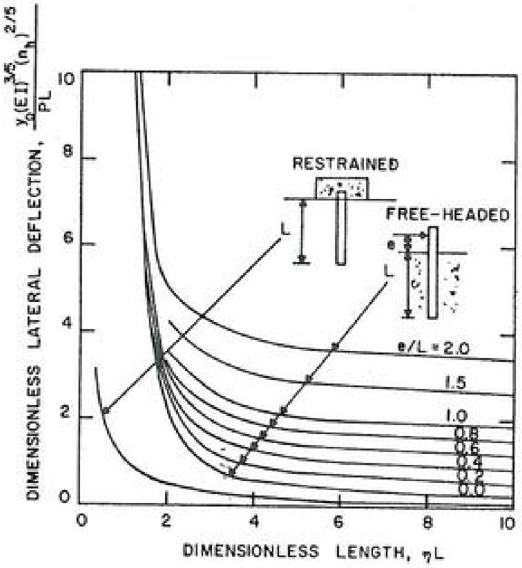


FIG. 3.—LATERAL DEFLECTIONS AT GROUND SURFACE

in which P is the applied lateral load at the ground surface, n_h is a coefficient which depends on the relative density of the soil surrounding the loaded pile (Table 1) and EI is the stiffness of the pile section.

Short Piles.—The lateral deflection y_o at the ground surface for a free-headed short pile (with a dimensionless length η L) less than 2.0) can be calculated directly from the equation:

$$y_o = \frac{18\ P\left(1 + 1.33\ \frac{e}{L}\right)}{L^2\ n_h} \quad \text{(3a)}$$

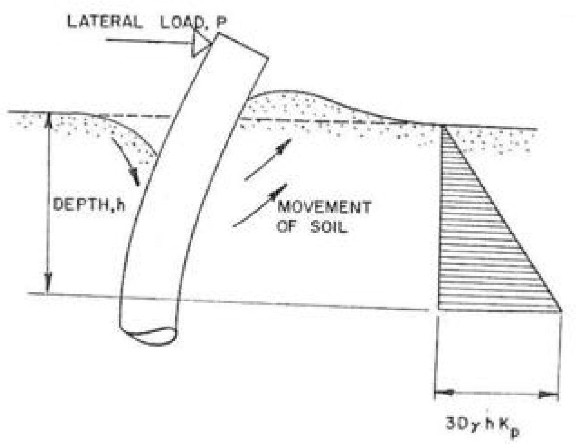
e is the eccentricity of the applied load P (Fig. 5). The corresponding lateral deflection for a short fixed-headed pile is equal to:

$$y_o = \frac{2\ P}{L^2\ n_h} \quad \text{(3b)}$$

in which L is the embedment length.

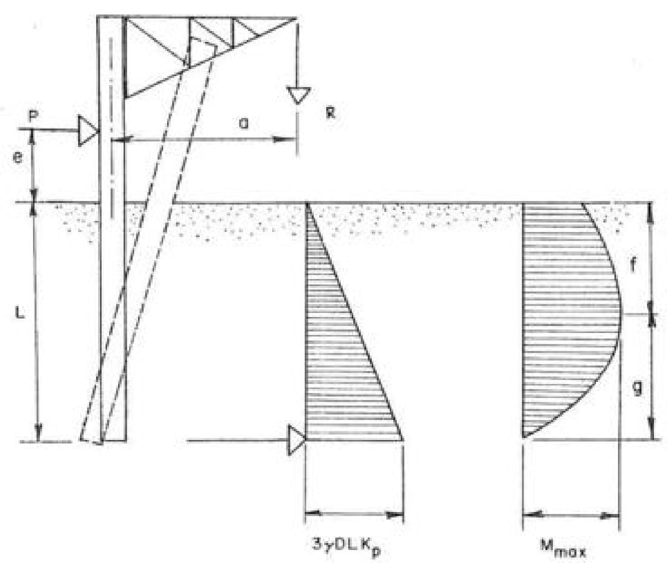
The lateral deflections can also be calculated for the case when the stiffness of the pile section varies laong the length of the pile.[44]

44 Reese, L. C., and Ginzbarg, A. S., "Step-Tapered Beams on Foundations Having Variable Stiffness," Publication No. 221, Shell Development Co., Exploration and Production Research Div., Houston, Tex., 1960.



(a) DEFLECTIONS (b) DISTRIBUTION OF SOIL REACTIONS

FIG. 4.—ASSUMED DISTRIBUTION OF SOIL REACTIONS



(a) DEFLECTIONS (b) SOIL REACTIONS (c) BENDING MOMENT

FIG. 5.—SHORT FREE-HEADED PILES—DISTRIBUTION OF DEFLECTIONS, SOIL REACTIONS, AND MOMENTS

The method of evaluating the deflections of laterally loaded piles described above is probably not applicable to very short piles, when the length of the laterally loaded piles is less than approximately four pile diameters due to edge effects at the bottom of the piles. No test data are available for very short piles. Therefore, lateral deflections of such piles can be estimated only from load tests.

Repetitive loads cause an increase of the lateral deflections at working loads. Prakash[21] has observed that the lateral deflections of a pile group embedded in a sand of medium relative density increased to twice the initial deflections after 40 load cycles. This increase corresponds to a decrease of the coefficient of subgrade reaction to one-third its initial value. No additional increase of the lateral deflections was observed at further increases of the number of load cycles. It is expected that the effects of load repetitions will be larger at low relative densities than at high relative densities of the surrounding soil. Therefore, it appears reasonable to evaluate the effect of repeated loads on lateral deflections at low relative densities by assuming that the coefficient of lateral subgrade reaction is one-fourth its initial value. For soils with high relative densities, the effect of load repetitions may be evaluated if the effective coefficient of subgrade reaction is taken as half its initial value. As the available test data are limited, the preceding values given should be used with caution.

Comparison with Test Data.—A number of lateral load tests have been carried out on free-headed and restrained steel, reinforced concrete and timber piles driven or jetted into dense, medium or loose cohesionless soils. The calculated lateral deflections depend on the stiffness of the pile sections, the relative density of the soil surrounding the test piles and on the degree of end restraint. Frequently insufficient information concerning these properties is available. When no data are available, the following assumptions have been made:

1. The moduli of elasticity for the pile materials steel, concrete and wood are 30×10^6 psi, 3×10^6 psi and 1.5×10^6 psi, respectively.
2. The numerical values of the coefficient n_h (Eq. 1) recommended by Karl Terzaghi[37] can be used. Terzaghi[37] recommended the values 7 tons per cu ft, 21 tons per cu ft and 56 tons per cu ft, respectively for cohesiveless soils of low, medium and high relative density when the ground water table is located at or below the significant depth and the values 4 tons per sq cu ft, 14 tons per sq cu ft and 34 tons per sq cu ft, respectively, when the ground water table is located at the ground surface.
3. The stiffness of concrete piles can be evaluated on the basis of its uncracked section neglecting the reinforcement.

The calculated lateral deflections have been compared in Table 2 with some available test data.[30,22,33,5,31,14,24,11,7,20,34,8,15,36,18,3,2,21] The test data are examined in detail in Appendix II.

In general, the relative densities of the supporting soils are difficult to estimate due to a lack of test data. Therefore, the lateral deflections have been calculated for two assumed values of the coefficient n_h. The assumed values of n_h are given also in Table 2. The test data (Table 2) indicate that pile groups behave similarly as single piles at working loads and that the total lateral resistance can be calculated as the sum of the resistances of the

TABLE 2.—LATERAL DEFLECTIONS

Test (1)	Applied Lateral Load per Pile P, in tons (2)	Measured Lateral Deflections y_{test}, in inches (3)	Calculated Lateral Deflections y_{calc}, in inches (4)	Ratio y_{calc}/y_{test} (5)
		(a) Sandeman[30], 1880		
6	20.0	4.0	7.30[b] 3.79[d]	1.82[b] 0.95[d]
		(b) Raes[22], 1936		
1[f]	2.85	4.72	0.57[a] 0.27[c]	0.12[a] 0.06[c]
2[f]	5.20	12.1	1.05[a] 0.49[c]	0.09[a] 0.04[c]
3[2)]	3.0	0.59	0.53[b] 0.28[d]	0.90[b] 0.48[d]
		(c) Vierhelier[33], 1937		
1	15.0	0.46	0.93[d] 0.52[e]	2.02[d] 1.13[e]
2	15.0	0.44	0.93[d] 0.52[e]	2.11[d] 1.18[e]
		(d) Feagin[5], 1937		
164 BB[f]	19.50	1.30	2.47[a] 1.17[c]	1.90[a] 0.90[d]
152 BE[f]	16.25	2.40	4.77[a] 2.25[c]	1.98[a] 0.94[c]
5 2	30.0	1.75	3.79[a] 1.79[c]	2.17[a] 1.02[c]
6 22	30.0	1.75	1.46[a] 0.69[c]	0.83[a] 0.40[c]
2 2	20.0	1.22	2.52[a] 1.19[c]	2.07[a] 0.98[c]
3 2	20.0	1.19	2.52[a] 1.19[c]	2.11[a] 1.00[c]
1 2	18.3	1.44	1.60[a] 0.76[c]	1.11[a] 0.53[c]
		(e) Shilts, Graves and Driscoll[31], 1948		
1	0.52	0.55	2.33[b] 0.78[d]	4.23[b] 1.42[d]
2	0.50	0.35	1.48[b] 0.49[d]	4.22[b] 1.40[d]
2[f]	0.50	0.10	0.90[b] 0.30[d]	9.00[b] 3.00[d]
6	0.50	0.25	2.31[b] 0.77[d]	9.23[b] 3.08[d]
17	0.010	0.10	0.45[b] 0.15[d]	4.50[b] 1.50[d]
18	0.022	0.23	0.45[b] 0.15[d]	1.96[b] 0.65[d]
19	0.032	0.17	0.37[b] 0.12[d]	2.19[b] 0.71[d]
20	0.035	0.18	0.39[b] 0.13[d]	2.17[b] 0.72[d]
21	0.021	0.21	0.24[b] 0.08[d]	1.14[b] 0.38[d]
22	0.024	0.22	0.49[b] 0.16[d]	2.23[b] 0.73[d]
23	0.023	0.23	0.47[b] 0.16[d]	2.05[b] 0.70[d]
25	0.030	0.25	0.61[b] 0.20[d]	2.44[b] 0.80[d]
28	0.016	0.10	1.14[b] 0.38[d]	11.4[b] 3.8[d]

TABLE 2.—*CONTINUED*

(1)	(2)	(3)	(4)	(5)
		(f) Loos and Breth[14], 1949		
1	0.110	0.13	0.28[b] 0.15[d]	2.15[b] 1.15[d]
2	0.110	0.10	0.28[b] 0.15[d]	2.80[b] 1.50[d]
3	0.110	0.06	0.28[b] 0.15[d]	4.67[b] 2.50[d]
4	0.085	0.03	0.16[b] 0.08[d]	5.33[b] 2.67[d]
5	0.095	0.04	0.20[b] 0.10[d]	5.00[b] 2.50[d]
6	0.101	0.04	0.22[b] 0.11[d]	5.50[b] 2.75[d]
7	0.010	0.05	0.05[b] 0.03[d]	1.00[b] 0.60[d]
8	0.044	0.07	0.20[b] 0.11[d]	2.85[b] 1.57[d]
		(g) Raymondi[24], 1950		
1	2.14	0.096	0.217[d] 0.121[e]	2.26[d] 1.26[e]
2	2.14	0.111	0.217[d] 0.121[e]	1.95[d] 1.09[e]
3	2.14	0.097	0.217[d] 0.121[e]	2.23[d] 1.25[e]
4	2.14	0.099	0.217[d] 0.121[e]	2.19[d] 1.22[e]
5	3.16	0.175	0.320[d] 0.178[e]	1.83[d] 1.02[e]
6	2.14	0.112	0.217[d] 0.121[e]	1.94[d] 1.08[e]
7	3.16	0.168	0.320[d] 0.178[e]	1.90[d] 1.06[e]
8	2.14	0.128	0.217[d] 0.121[e]	1.70[d] 0.95[e]
9	3.16	0.179	0.320[d] 0.178[e]	1.78[d] 0.99[e]
10	2.14	0.117	0.217[d] 0.121[e]	1.85[d] 1.03[e]
11	3.16	0.171	0.320[d] 0.178[e]	1.87[d] 1.04[e]
12	2.14	0.124	0.217[d] 0.121[e]	1.75[d] 0.98[e]
13	3.16	0.169	0.320[d] 0.178[e]	1.89[d] 1.05[e]
		(h) Knopp[11], 1952		
	4.4	0.67	2.33[a] 0.67[c]	3.48[a] 1.00[c]
		(i) Feagin[7], 1953		
3	7.5	0.492	0.73[b] 0.38[d]	1.48[b] 0.77[d]
2	12.0	0.790	1.18[b] 0.61[d]	1.50[b] 0.77[d]
		(j) O'Halloran[20], 1953		
	22.5	1.63	2.00[b] 1.04[d]	1.23[b] 0.64[d]
		(k) Wagner[34], 1953		
15	4.0	0.46	0.82[a] 0.39[c]	1.78[a] 0.85[c]
25	4.0	0.22	0.64[a] 0.30[c]	2.91[a] 1.36[c]

TABLE 2.—*CONTINUED*

(1)	(2)	(3)	(4)	(5)
(m) Gleser[8], 1953				
A-446	9.67	1.34	2.47[b] 1.28[d]	1.84[b] 0.96[d]
(n) Mason and Bishop[15], 1953				
	9.25	1.35	2.29[b] 1.18[d]	1.69[b] 0.87[d]
(o) Wen[36], 1955				
T4	0.050	0.150	0.125[b] 0.065[d]	0.83[b] 0.43[d]
T8	0.033	0.040	0.083[b] 0.043[d]	2.07[b] 1.07[d]
(p) McNulty[18], 1956				
1	5.0	0.05	0.56[b] 0.29[d]	11.2[b] 5.8[d]
2	11.5	0.80	1.70[b] 0.88[d]	2.13[b] 1.10[d]
3	11.5	0.55	2.07[b] 1.07[d]	3.77[b] 1.94[d]
(q) California Division of Highways[3], 1958				
I16	11.0	0.520	1.51[b] 0.80[d]	2.90[b] 1.54[d]
I36N	42.5	0.535	1.54[b] 0.81[d]	2.87[b] 1.51[d]
I36S	42.5	0.385	1.54[b] 0.81[d]	4.00[b] 2.10[d]
I36C	13.8	0.490	0.68[b] 0.36[d]	1.39[b] 0.74[d]
(r) Broms[2], 1960				
1	20.0	0.55	1.10[b] 0.57[d]	2.00[b] 1.04[d]
2	16.0	1.15	2.27[b] 1.10[d]	1.97[b] 0.96[d]
(s) Prakash[21], 1962				
1	0.0026	0.024	0.103[d] 0.057[e]	4.28[d] 2.37[e]
3	0.0024	0.023	0.037[d] 0.020[e]	1.61[d] 0.87[e]
17	0.0017	0.010	0.025[d] 0.015[e]	2.60[d] 1.50[e]
4	0.0019	0.009	0.030[d] 0.016[e]	3.33[d] 1.78[e]
5	0.0019	0.008	0.030[d] 0.016[e]	3.75[d] 2.00[e]
6	0.0023	0.011	0.035[d] 0.019[e]	3.19[d] 1.73[e]
7	0.0029	0.016	0.044[d] 0.024[e]	2.75[d] 1.50[e]
13	0.0006	0.008	0.009[d] 0.005[e]	1.12[d] 0.63[e]
16	0.0011	0.008	0.017[d] 0.009[e]	2.13[d] 1.13[e]
19	0.0021	0.025	0.083[d] 0.046[e]	3.33[d] 1.83[e]
9	0.0023	0.009	0.035[d] 0.019[e]	3.89[d] 2.11[e]
10	0.0023	0.007	0.035[d] 0.019[e]	5.00[d] 2.71[e]
11	0.0023	0.009	0.035[d] 0.019[e]	3.89[d] 2.11[e]

TABLE 2.—*CONTINUED*

(1)	(2)	(3)	(4)	(5)
(s) Prakash[21], 1962 (Continued)				
24	0.0019	0.018	0.030[d] 0.016[e]	1.67[d] 0.89[e]
21	0.0020	0.015	0.031[d] 0.017[e]	2.07[d] 1.13[e]
22	0.0017	0.013	0.026[d] 0.015[e]	2.00[d] 1.15[e]
23	0.0017	0.011	0.026[d] 0.015[e]	2.47[d] 1.36[e]

[a] Assumed n_h = 4.0 tons per cu ft
[b] Assumed n_h = 7.0
[c] Assumed n_h = 14.0
[d] Assumed n_h = 21.0
[e] Assumed n_h = 56.0
[f] Piles jetted
[g] At 42% of ultimate lateral resistance

individual piles. The test data indicate further that the lateral resistance of jetted piles at a given lateral deflection may only be a fraction of the lateral resistance of driven piles.

It can be seen further that the calculated lateral deflections in almost all cases exceeded considerably the measured lateral deflections. This observation was made for single piles as well as for pile groups, for unrestrained piles as well as for restrained piles. The conclusion may be drawn that the values recommended by Terzaghi will in general over-estimate at working loads the deflections at the ground surface, and thus will yield results which are on the safe side except for piles which have been placed with the aid of jetting.

ULTIMATE LATERAL RESISTANCE

General.—The mode of failure and the resulting distribution of lateral earth pressures are shown in Fig. 4 for a laterally loaded pile driven into a cohesionless soil. At failure, the soil located in front of the pile moves in an upward direction, whereas the soil located at the back side of the pile moves downward and fills the void created by the lateral deflection of the pile. However, at relatively large depths, the soil located in front of the pile will move laterally to the back side of the pile instead of upwards. Approximate calculations have indicated that down to a depth of approximately fifty pile diameters, the lateral resistance with respect to an upward movement of the soil is less than the resistance with respect to a lateral movement. This critical depth of fifty pile diameters corresponds to an angle of internal friction of approximately 30°.

Close to the ground surface, passive lateral earth pressures develop at the front face of the loaded piles while active pressures develop at the back face as soon as the rotation of the pile exceeds approximately 0.002 and 0.006 radians for a dense and loose sand, respectively.[45] (It has been assumed that this rotation is approximately equal to that required to develop the passive pressure against a long wall.) The lateral earth pressures can be calculated by standard earth pressure theories within a depth of approximately one pile diameter below the ground surface assuming that the pile is infinitely wide.

If the surface of the pile is assumed frictionless, the lateral earth pressures can be calculated by the Rankine earth pressure theory. Below a depth of approximately one pile diameter, the soil reactions which develop at the front face are larger than the passive lateral resistance corresponding to an infinitely long wall due to lateral stress distribution in the soil. The corresponding active lateral resistance acting on the back face will be less than that calculated by standard earth pressure theories.

In the following analysis, the active earth pressure has been assumed small compared to the passive earth pressure and has been neglected in the calculations. Driving of piles cause an increase of the relative density of the soil surrounding the piles to a distance of approximately one diameter from the pile surface. This increase of the relative density results in a lateral soil reaction larger than that estimated from standard penetration or laboratory triaxial or direct shear tests.

Laboratory tests by Shilts, Graves and Driscoll,[31] by Ramelot and Vandeperre,[23] and by Roscoe[28] with model piles of different shapes have indicated that the shape of the pile section has only a small effect on the ultimate lateral resistance and on the lateral earth pressure distribution. Prakash[21] has observed that the maximum lateral earth pressure is two to three times the passive earth pressure calculated by the Rankine earth pressure theory.

In the following analysis, it will be assumed that the lateral earth pressure which develops at failure is equal to three times the passive Rankine earth pressure and that this lateral earth pressure is independent of the shape of the cross-sectional area of the laterally loaded pile. The accuracy of this assumption can be established only from comparisons with test data. Such comparisons will be made subsequently. It will be shown that this assumption will yield results which are on the safe side.

The assumed distribution of lateral earth pressure at failure is shown in Fig. 4 (b). At the depth z below the ground surface, the assumed soil reaction Q per unit length of the pile will be:

$$Q = 3D \gamma z K_p \quad \ldots\ldots\ldots\ldots \quad (4)$$

in which D is the pile diameter, γ is the unit weight of the soil and K_p is the coefficient of passive earth pressure as calculated by the Rankine earth pressure theory. The unit weight γ is equal to the submerged unit weight if the ground water table is located at or above the ground surface and is equal to the wet unit weight if the ground water table is located below the section considered.

45 Sowers, G. B. and Sowers, G. F., "Introductory Soil Mechanics and Foundations," The MacMillan Co., New York, N. Y., 1961, 2nd Edition.

For a cohesionless soil, the coefficient K_p can be calculated from

$$K_p = \frac{1 + \sin \phi_d}{1 - \sin \phi_d} \quad \ldots\ldots\ldots\ldots \quad (5)$$

in which ϕ_d is the estimated or measured angle of internal friction as determined from drained triaxial or direct shear tests.

Short Free-Headed Pile.—The mode of failure of a laterally loaded pile driven into a cohesionless soil will depend on the depth of embedment and on the degree of end restraint. The mode of failure of a short free-headed pile is shown in Fig. 5. Failure takes place when the pile rotates as a unit around a point located below the ground surface. The corresponding assumed distributions of lateral earth pressures and bending moments are shown in Figs. 5 (b) and 5 (c), respectively. It has been assumed that the lateral deflections are sufficiently large, at failure, as to develop the full passive resistance equal to three times the passive Rankine earth pressure from the ground surface down to the location of the center of rotation.

High negative earth pressures develop close to the toe of the laterally loaded pile and it has been assumed for the purpose of analysis that this pressure can be replaced by a concentrated load as shown in Fig. 5 (b).

The ultimate lateral resistance can then be evaluated from equilibrium requirements. The driving moment M_d caused by the applied forces P and R with respect to the toe of the loaded pile is

$$M_d = P (e + L) + Ra \quad \ldots\ldots\ldots\ldots \quad (6)$$

The corresponding moment at failure caused by the lateral earth pressures acting along the embedded portion of the pile M_r can be calculated from

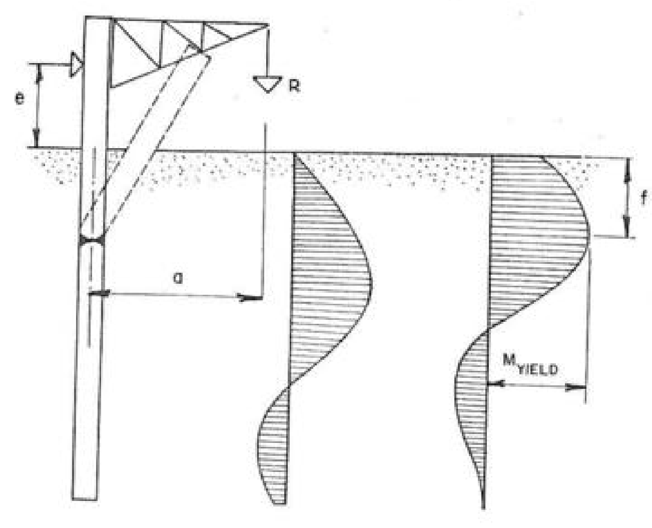
$$M_r = 0.5 \gamma D L^3 K_p \quad \ldots\ldots\ldots\ldots \quad (7)$$

Equilibrium requires that the driving moment M_d is, at failure, equal to the resisting moment M_r. The ultimate lateral resistance P can then be evaluated from Eqs. 6 and 7. For the case when the applied load R is equal to zero, the lateral resistance P is

$$P = \frac{0.5 \gamma D L^3 K_p}{(e + L)} \quad \ldots\ldots\ldots\ldots \quad (8)$$

For the case when the lateral load P is equal to zero, the load R causing failure can be evaluated from

$$R = \frac{0.5 \gamma D L^3 K_p}{a} \quad \ldots\ldots\ldots\ldots \quad (9)$$



(a) DEFLECTION (b) SOIL REACTION (c) BENDING MOMENT

FIG. 6.—LONG FREE-HEADED PILES—DISTRIBUTION OF DEFLECTIONS, SOIL REACTIONS, AND MOMENTS

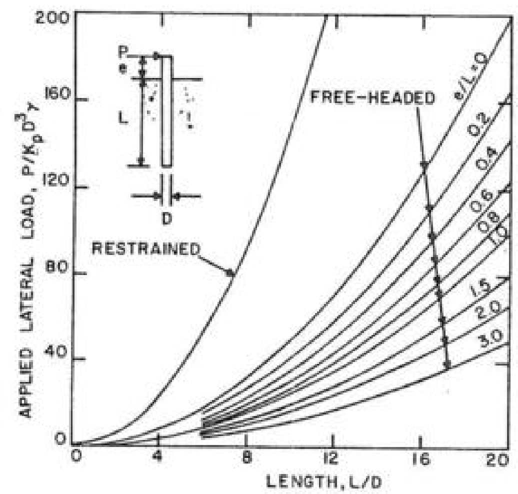


FIG. 7.—ULTIMATE LATERAL RESISTANCE—SHORT PILES

In the preceding analysis, it has been assumed that the maximum positive bending moment in the laterally loaded pile which develops at the distance f below the ground surface (Fig. 5) is less than the ultimate or yield resistance of the pile section, M_{yield}.

The dimensionless ultimate lateral resistance $P/K_p \gamma D^3$ determined from Eq. 8 has been plotted in Fig. 7 as a function of the dimensionless embedment length L/D. It should be noticed that at small eccentricity ratios e/D, the ultimate lateral resistance increases linearly with $(L/D)^2$.

Long Free-Headed Pile.—The mode of failure of a long free-headed pile is shown in Fig. 6. Failure takes place when a plastic hinge forms a distance f below the ground surface (at the location of the maximum bending moment). The maximum bending moment M_{max}^{pos} and the distance f can be evaluated if it is assumed that the passive lateral earth pressure (equal to approximately three times the passive Rankine earth pressure) develops from the ground surface down to the location of the maximum bending moment. The distance f can be determined from the requirement that the total shear force at the depth f is equal to zero, then

$$f = 0.82\sqrt{\frac{P}{\gamma D K_p}} \quad \ldots\ldots\ldots\ldots\ldots (10)$$

The corresponding maximum positive bending moment M_{max}^{pos} can then be determined from

$$M_{max}^{pos} = P\,(e + 0.67\,f) + Q\,a \quad \ldots\ldots\ldots\ldots (11)$$

Failure takes place when the maximum bending moment M_{max}^{pos} is equal to the ultimate or yield resistance of the pile section M_{yield}. For the case when the applied load Q is equal to zero, the ultimate lateral resistance P can be calculated from

$$P = \frac{M_{yield}}{\left(e + 0.54\sqrt{\frac{P}{\gamma D K_p}}\right)} \quad \ldots\ldots\ldots\ldots (12)$$

The dimensionless ultimate lateral resistance $P/K_p \gamma D^3$ has been plotted in Fig. 8 as a function of the dimensionless ultimate or yield resistance $M_{yield}/D^4 \gamma K_p$ and the eccentricity ratio e/D. It can be seen that the ultimate resistance increases rapidly with increasing yield resistance M_{yield}.

Short Restrained Pile.—The lateral deflections and distribution of soil reactions and bending moments for a short restrained pile is shown in Fig. 9. Failure takes place when the load applied to the pile is equal to the ultimate lateral resistance of the soil as expressed by

$$P = 1.5\,\gamma\,L^2\,D\,K_p \quad \ldots\ldots\ldots\ldots\ldots (13)$$

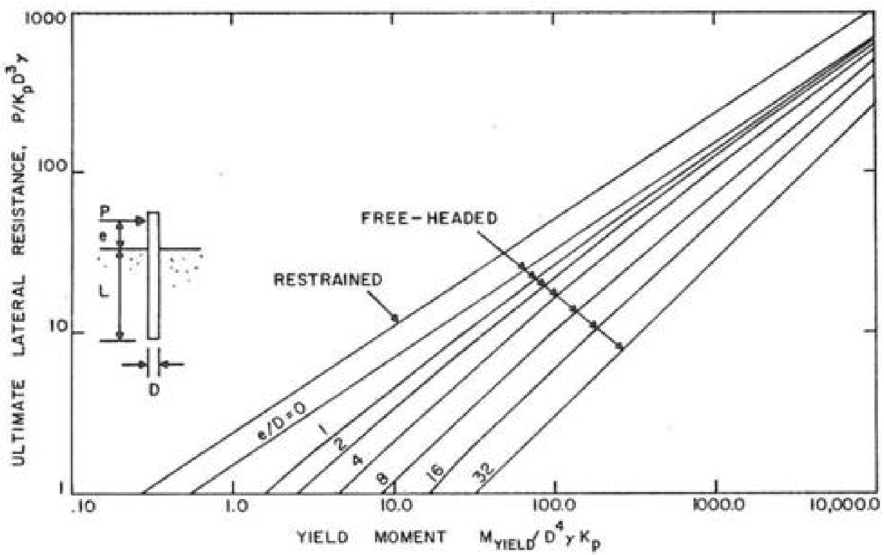


FIG. 8.—ULTIMATE LATERAL RESISTANCE—LONG PILES

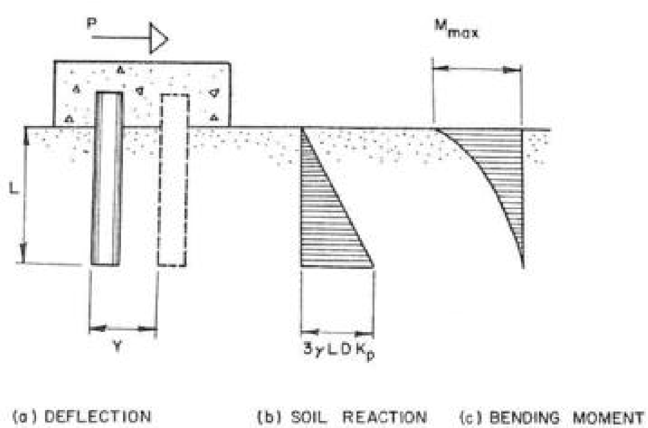


FIG. 9.—SHORT RESTRAINED PILES—DISTRIBUTION OF DEFLECTIONS, SOIL REACTIONS, AND MOMENTS

The dimensionless ultimate lateral resistance $P/K_p \gamma D^3$ as determined from Eq. 13 has been plotted in Fig. 7 as a function of the embedment length L/D. Eq. 13 is only applicable when the maximum negative bending moment (which develops at the pile cap or at a restraining bracing system) is less than the ultimate or yield resistance M_{yield} of the pile section.

Intermediate Length Restrained Pile.—The mode of failure for a restrained pile of intermediate length is illustrated in Fig. 10. Failure takes place when the maximum negative moment at the head of the pile reaches the yield resistance of the pile section. The ultimate lateral resistance can then be calculated assuming that the lateral earth pressure increases linearly with depth and that its intensity is equal to three times the passive Rankine earth pressure. The high negative lateral earth pressure acting close to the toe of the lateral pile has been approximated by a concentrated force as shown in Fig. 10 (b). The ultimate lateral resistance P can then be determined from equilibrium considerations with respect to the toe of the loaded pile as expressed by

$$P = 0.5\,\gamma\, D\, L^2\, K_p - M_{yield} \qquad (14)$$

The dimensionless ultimate lateral resistance $P/K_p \gamma D^3$ as determined from Eq. 14 is shown in Fig. 7 as a function of the dimensionless embedment length L/D. It should be noticed that the ultimate lateral resistance can be determined from Eq. 14 only when the positive bending moment which develops at the depth f below the ground surface is less than the ultimate or yield resistance of the pile section.

Long Restrained Pile.—The mode of failure of a long laterally restrained pile is shown in Fig. 11. Failure takes place when two plastic hinges form and the pile turns into a mechanism. The two plastic hinges form when the maximum positive bending moment at the depth f below the ground surface and the maximum negative bending moment at the bottom of the pile cap or lateral bracing system both reach the yield resistance of the pile section.

The distance f (the location of the maximum positive bending moment) can be determined from the consideration that the total shear force at this depth is equal to zero and that the full passive lateral soil pressure develop along the embedded portion of the pile down to the depth f. This depth can be evaluated directly from Eq. 10. The ultimate lateral resistance can then be calculated as

$$P = \frac{M^{pos}_{yield} + M^{neg}_{yield}}{e + 0.54\sqrt{\dfrac{P}{\gamma\, D\, K_p}}} \qquad (15a)$$

where M^{pos}_{yield} and M^{neg}_{yield} are the yield resistances of the pile sections located at the maximum positive and maximum negative bending moments, respectively.

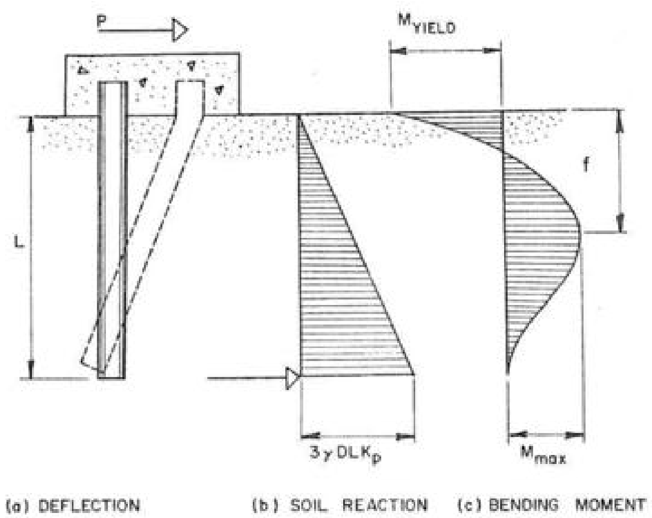


FIG. 10.—INTERMEDIATE LONG RESTRAINED PILES—DISTRIBUTION OF DEFLECTIONS, SOIL REACTIONS, NAD MOMENTS

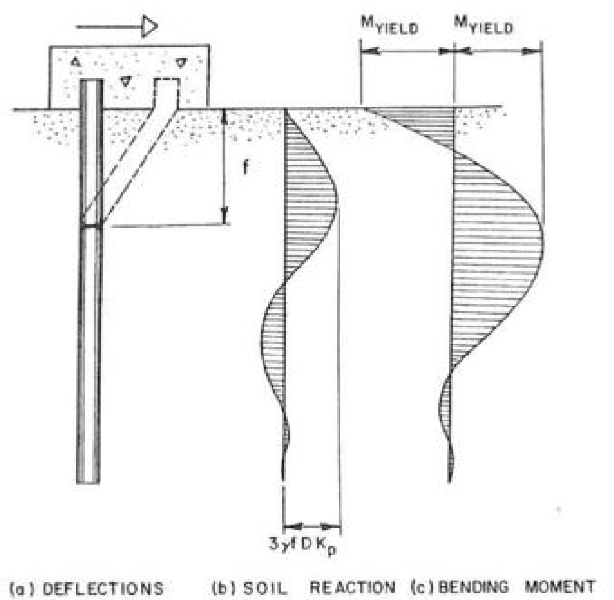


FIG. 11.—LONG RESTRAINED PILES—DISTRIBUTION OF DEFLECTIONS, SOIL REACTIONS, AND MOMENTS

When the yield resistance of the pile sections located at the maximum positive moment and the maximum negative moments are equal, then Eq. 15a simplifies to

$$P = \frac{2\ M_{yield}}{e + 0.54\sqrt{\frac{P}{\gamma\ D\ K_p}}} \qquad \text{(15b)}$$

The ultimate lateral resistance of a restrained long pile is considerably higher than the ultimate lateral resistance of a corresponding free-headed pile. The ultimate lateral resistance as determined from Eq. 15b is shown in Fig. 8 as a function of the dimensionless yield or plastic moment resistance $M_{yield}/\gamma\ D^4K_p$.

The ultimate lateral resistance of a pile group may be considerably less than the sum of the ultimate lateral resistance of the individual piles. The soil located within a pile group will move as a unit with the laterally loaded piles when the spacing between the individual piles is small. To the knowledge of the author, no data of the ultimate lateral resistance of pile groups are available.

Test Data.—The ultimate lateral resistances as calculated by Eqs. 8, 9, 12, 13, 14, 15a or 15b have been compared in Table 3 with some available test data.[23,20,32,35,28,14,2] The test data are considered in detail in Appendix II.

The average measured ultimate lateral resistances exceeded the calculated resistances by more than 50 percent. However, the measured ultimate lateral resistances of piles 4 and 5 tested by Walsenko[35] were found to be only two-thirds of the calculated resistances. These piles were embedded in a fine gravel with a reported angle of internal friction of 45° as measured by direct shear tests. Frequently it is difficult to measure accurately the shearing strength of gravel by means of direct shear tests. If a value of 35° is taken as the angle of internal friction, then the average ratio P_{test}/P_{calc} is increased to 1.43, a value which compares well with the remainder of the test data.

The conclusion can be drawn from this comparison that the ultimate resistance of laterally baded piles can be estimated conservatively assuming an ultimate lateral soil reaction equal to three times the Rankine lateral earth pressure.

The available test data covers a wide range of soil conditions as well as a wide range of pile types. Timber, wood and steel piles have been tested. The diameter and the length of the test piles varied from 1.5 in. to 15.0 in. and from 1.64 ft to 40 ft, respectively. In general, sufficient information was not available for a complete analysis of the test data and frequently the strength properties of the pile material or of the supporting soil had to be estimated. Additional data are required before complete confidence can be gained in the proposed methods.

TABLE 3.—ULTIMATE LATERAL RESISTANCE

Test	Depth of Embedment L, in feet	Eccentricity e, in feet	Measured Ultimate Lateral Resistance, P_{test}, in kips	Calculated Ultimate Lateral Resistance, P_{calc}, in kips	Ratio P_{test}/P_{calc}
(a) Ramelot and Vandeperre[23], 1950					
1a	1.64	3.28[a]	0.153	0.072	2.13
1b	1.97	3.28	0.300	0.126	2.38
1c	2.29	3.28	0.437	0.184	2.37
2a	1.64	6.55	0.138	0.108	1.28
2b	1.97	6.55	0.225	0.188	1.20
2c	2.29	6.55	0.390	0.303	1.28
2d	2.63	6.55	0.588	0.455	1.29
3a	1.64	3.28	0.057	0.019	3.00
3b	1.97	3.28	0.110	0.037	2.97
3c	2.29	3.28	0.205	0.064	3.20
3d	2.62	3.28	0.305	0.101	3.03
4a	1.64	6.55	0.100	0.036	2.77
4b	1.97	6.55	0.193	0.063	3.06
4c	2.29	6.55	0.275	0.092	2.99
4d	2.62	6.55	0.450	0.146	3.09
4e	2.95	6.55	0.592	0.207	2.85
5a	1.64	3.28	0.165	0.072	2.29
5b	1.97	3.28	0.278	0.126	2.15
5c	2.29	3.28	0.422	0.184	2.29
(b) O'Halloran[20], 1953					
	21.0	—	106.0	70.3	1.51
(c) Tschebotarioff[32], 1953					
1	2.42	3.75	0.042	0.0197	2.13
2	2.42	3.75	0.042	0.0197	2.13
(d) Walsenko[35], 1955					
1	2.0	11.13	0.200	0.112	1.78
2	2.5	10.64	0.400	0.189	2.11
3	3.0	10.16	0.400	0.295	1.35
4	3.0	10.16	0.100	0.159	0.63
5	4.0	9.20	0.260	0.378	0.69
(e) Roscoe[28], 1957					
1	0.75	1.90	0.0051	0.0051	1.00
2	0.75	1.90	0.0048	0.0051	0.94
3	0.75	1.90	0.0037	0.0036	1.03
(f) Loos and Breth[14], 1949					
	4.92	1.64	0.084	0.070	1.20
(g) Broms[2], 1960					
	53.5	0.6	32.0	18.1	1.77

[a] Length of cantilever arm

MOMENT DISTRIBUTION

General.—The distribution of bending moment for a laterally loaded free-headed or restrained pile depends on the degree of end restraint, on the ultimate or yield resistance of the pile section, and on the shear strength and deformation characteristics of the supporting soil.

Free-Headed Piles.—The distribution of bending moments for a laterally loaded free-headed pile is shown in Figs. 5 and 6. The maximum positive bending moment M^{pos}_{max} occurs at the depth f below the ground surface. The maximum bending moment M^{pos}_{max} as well as the distance f can be calculated if the lateral soil reactions down to the depth f below the ground surface can be estimated. It will be assumed in the following analysis that lateral earth pressure is equal to three times the Rankine passive pressures. (The accuracy and the limitations of this assumption can only be established from comparisons with test data. The distance f and the maximum bending moment can then be calculated from Eqs. 10 and 11, respectively.

Restrained Piles.—The distribution of bending moment in a short laterally loaded restrained pile is shown in Fig. 9. The maximum negative bending moment M^{neg}_{max} which will occur at the bottom of the pile cap is equal to

$$M^{neg}_{max} = 0.67 \ P \ L \quad \ldots\ldots\ldots\ldots\ldots \quad (16)$$

Eq. 16 is only applicable as long as the maximum negative bending moment does not exceed the ultimate or yield resistance of the pile section.

The distribution of bending moment for intermediate and long restrained laterally loaded piles depends on the degree of end restraint. The maximum positive banding moment occurs at a depth f below the ground surface. The depth f can be determined from Eq. 10. The positive moment M^{pos}_{max} can then be calculated from

$$M^{pos}_{max} = P \ (e + 0.67 \ f) - M^{neg}_{max} \quad \ldots\ldots\ldots\ldots \quad (17)$$

In this equation M^{neg}_{max} is the restraining negative moment which occurs at the bottom of the pile cap. The maximum restraining moment depends mainly on the degree of end restraint and may vary between wide limits. It should, however, be noticed that the sum of the maximum positive and negative bending moment is independent of the degree of end restraint and that this sum is a function of the applied lateral load, of the dimensions of the test pile, of the unit weight and of the strength properties of the supporting soil. The sum of the maximum positive and negative bending moment can be determined from

$$M^{pos}_{max} + M^{neg}_{max} = \frac{0.54 \ P^{3/2}}{\left(\gamma \ D \ K_p\right)^{1/2}} \quad \ldots\ldots\ldots\ldots \quad (18)$$

Repetitive loads affect the magnitude and distribution of bending moments. Prakash[21] has observed that the maximum positive bending moment for a

TABLE 4.—MAXIMUM BENDING MOMENT

Test	Depth of Embedment L, in feet	Eccentricity e, in feet	Applied Load P, in kips	Measured Maximum Bending Moment M_{test}, in kip-feet	Calculated Maximum Bending Moment M_{calc}, in kip-feet	Ratio M_{test}/M_{calc}
(a) Loos and Breth[14], 1949						
1	4.92	0	0.220	0.160	0.184	0.87
2	3.28	0	0.220	0.165	0.184	0.90
3	2.46	0	0.220	0.115	0.184	0.62
4	4.92	1.64	0.170[a]	0.125	0.124	1.00
5	3.28	3.28	0.190[a]	0.160	0.148	1.08
6	2.46	4.09	0.201[a]	0.125	0.161	0.78
7	4.92	1.64	0.084[a]	0.200	0.185	1.08
8	4.92	1.64	0.110	0.230	0.238	0.97
(b) Wen[36], 1955						
T-8	3.25	0.5	0.100	0.065	0.077	0.84
(c) Roscoe and Schrofield[26], 1956						
7 X A	3.0	0	—	20.8	20.9	1.00
7 X E	3.0	0	—	15.6	20.9	0.75
7 Y A	3.0	0	—	19.8	20.9	0.95
7 Y E	3.0	0	—	24.4	20.9	1.16
(d) California Division of Highways[3], 1958						
I 16	30.0	-0.5	22.0	52	61	0.85
I 36N	45.0	-0.5	42.5	330	345	0.96
I 36S	35.0	-0.5	42.5	422	345	1.22
I 16C	30.0	-1.0	27.5	86	74	1.16
III 16	21.0	8.0	3.2	30	30	1.00
III 36N	36.0	8.0	40.0	346	444	0.78
III 36S	26.0	8.0	40.0	371	444	0.84
III 16C	21.0	8.0	8.6	75	86	0.87
III 36CN	36.0	8.0	33.0	377	357	1.05
III 36CS	26.0	8.0	52.0	542	601	0.90
(e) Prakash[21], 1962						
1	1.75	0	0.0053	0.00112	0.00120	0.93
3	1.75	0	0.0048	0.00127	0.00106	1.20
17	1.75	0	0.0033	0.00085	0.00060	1.42
4	1.75	0	0.0038	0.00094	0.00075	1.25
5	1.75	0	0.0038	0.00101	0.00075	1.35
13	1.75	0	0.0012	0.00019	0.00014	1.36
16	1.75	0	0.0022	0.00057	0.00036	1.58
19	1.75	0	0.0043	0.00083	0.00098	0.85
8	1.75	0	0.0046	0.00118	0.00097	1.22
9	1.75	0	0.0028	0.00064	0.00047	1.36
10	1.75	0	0.0046	0.00100	0.00097	1.03
11	1.75	0	0.0046	0.00135	0.00097	1.39
24	1.75	0	0.0038	0.00071	0.00072	0.99
21	1.75	0	0.0040	0.00122	0.00081	1.50
22	1.75	0	0.0035	0.00120	0.00065	1.85
23	1.75	0	0.0035	0.00111	0.00065	1.71

[a] Net force at the ground surface

free-headed and a restrained pile increased by 10% to 20% when subjected to 100 load cycles. It is, however, expected that the effects of repetitive loads will be larger in the case of a sand with a low relative density than for a sand with a high relative density.

Comparison with Test Data.—The distribution of bending moments in free-headed and restrained piles has been investigated by a number of investigators. The maximum bending moments as determined from Eqs. 11, 16, 17 or 18 are compared in Table 4 with test data as reported by Loos and Breth,[14] by Wen,[36] by Roscoe and Schofield,[26] by California Division of Highways,[3] and by Prakash.[21]

It can be seen from Table 4 that the measured maximum bending moments frequently exceeded the calculated values. The difference between measured and calculated values increased in general with increasing size of the pile group. The ratio M_{test}/M_{calc} has been plotted in Fig. 12 as a function of the

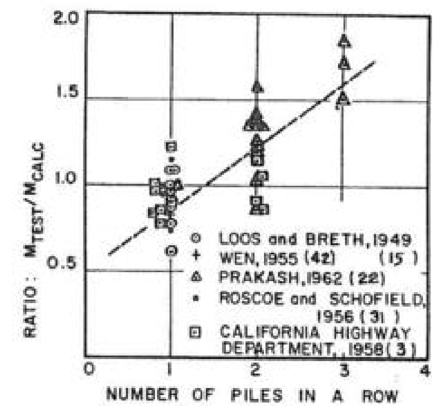


FIG. 12.—BEHAVIOR OF PILE GROUPS

number of piles in the direction of loading for each group. It can be seen from Fig. 12 that the maximum calculated bending moments agreed closely with the measured values for single piles, whereas the measured average maximum bending moment for pile groups consisting of three piles in the direction of loading exceeded the calculated maximum bending moment by approximately 70%.

This behavior is reasonable since the first row of piles in a pile group will cause a reduction of the lateral soil reactions acting on the following rows. Consequently, the average lateral earth pressure will be less than that assumed and the maximum average bending moment will be larger than that calculated.

The measured bending moments were found to be affected by the pile spacing. Due to group action, the measured bending moments exceeded the calculated bending moments when the spacing of the piles was less than about three pile diameters. At large pile spacings, the difference between measured and calculated maximum bending moments was small.

SUMMARY

1. The deflections of laterally loaded piles have been calculated assuming that the lateral earth pressures acting on the loaded piles increase linearly with increasing lateral deflection, that the corresponding coefficient of lateral subgrade reaction k_h increases linearly with depth and that its value can be estimated by a method proposed by Terzaghi.[39] A comparison with test data (Table 2) indicates that the calculated lateral deflection using the proposed values of the coefficient of subgrade reaction consistently exceeds the measured lateral deflections. Thus the proposed method yields values which are in general on the safe side. The discrepancy between measured and calculated values can be attributed at least partly to the increase in relative density of the soil which takes place during driving of the piles.

2. The ultimate lateral resistance of laterally loaded piles has been calculated assuming that the lateral soil reactions are at failure, three times the passive Rankine lateral pressure. Comparisons with test data indicate that the measured ultimate lateral resistances (Table 3) exceed the calculated resistances by approximately 50%. Thus the proposed method predicts consistently an ultimate lateral resistance which is less than the actual resistance.

3. The maximum and negative bending moments in free-headed and restrained laterally loaded piles has been calculated assuming that the lateral earth pressure above the location of the positive bending moment is equal to three times the passive Rankine earth pressure. Comparisons with test data (Table 4) indicate that the proposed method predicts closely the maximum positive and negative bending moments for single piles while for pile groups, the measured maximum bending moment exceeds the calculated maximum bending moment by approximately 70%.

APPENDIX I.—NOTATIONS

The following notations have been used in this paper:

a = moment arm (Figs. 5 and 6), in inches;

D = diameter or width of pile, in inches;

E = modulus of elasticity of pile material, in pounds per square inch;

e = eccentricity of applied load, in inches;

f = distance from ground surface to location of maximum bending moment, in inches;

I = moment of inertia of pile section, in.^{4};

K_p = coefficient of passive lateral earth pressure;

k_h = coefficient of horizontal subgrade reaction, in pounds per cubic inch;

L = length of embedment, in inches;

M = bending moment, in pound-inches;

M_{max}^{pos} = maximum positive bending moment, in pound-inches;

M_{max}^{neg} = maximum negative bending moment, in pound-inches;

M_{yield} = yield moment of pile section, in pound-inches;

M_{yield}^{pos} = yield resistance of pile section at location of maximum positive bending moment, in pound-inches;

M_{yield}^{neg} = yield resistance of pile section at location of maximum negative bending moment, in pound-inches;

n_h = coefficient of subgrade reaction for a long strip with a width of unity at a depth of unity, in pounds per cubic inch;

P = applied lateral load, in pounds;

Q = lateral soil reaction per unit length of pile, in pounds per foot;

R = applied axial load, in pounds;

y = lateral deflection, in inches;

y_o = lateral deflection at ground surface;

y_{calc} = calculated lateral deflections at ground surface, in inches;

y_{test} = measured lateral deflections at ground surface, in inches;

z = depth below ground surface, in inches;

ηL = dimensionless length;

γ = unit weight of soil, in pounds per cubic inch; and

ϕ_d = angle of internal friction of soil as measured by drained direct shear or triaxial tests, in degrees.

APPENDIX II.—COMPARISON WITH TEST DATA

The investigations referred to in Tables 2, 3 and 4 are considered in this Appendix.

Sandeman,[30] *1880.*—Sandeman measured the lateral resistance of a timber pile with a square cross-sectional area (12.5 in. by 12.5 in.). The pile was driven 14 ft into a sand. The lateral load was applied 6 in. above the ground surface. The dimensionless length η L was larger than 4.0. The lateral deflections have been calculated by Eq. 2a, assuming that the modulus of elasticity of the pile material was 1.5×10^6 psi and that the relative density of the sand was low to medium as indicated by the driving resistance of the test pile. Good agreement was found between the measured and calculated lateral deflection (Table 2).

Raes,[22] *1936*.—The lateral resistance of three reinforced concrete piles was measured by Raes. Test piles 1 and 2, with square cross-sectional area (13.8 in. by 13.8 in.) were placed by jetting in sand. At the site for test piles 1 and 2 the ground water table was located at the ground surface. Test pile 3 with a rectangular cross-sectional area (11.8 in. by 12.2 in.) was probably also placed by jetting. The ground water table was located 10 ft below the ground surface at the test site for pile 3. The lateral deflections of the piles have been calculated from Eq. 2a, assuming that the modulus of elasticity of the pile material was 3×10^6 psi. The lateral deflections of test piles 1 and 2 were calculated at the maximum applied load since the lateral deflections increased approximately linearly with load up to the maximum applied load. The lateral deflections for test pile 3 was calculated at the applied load of 3.0 tons since the lateral deflections increased approximately linear up to this load level. As expected the measured lateral deflections for the jetted piles 1 and 2 exceeded considerably the calculated values (Table 2).

Verhelier,[33] *1937*.—Verhelier investigated the lateral resistance of two foundation blocks, each supported by four vertical steel H-piles (Carnegie-Illinois 103) driven 30 ft through sand and gravel down to bed rock. The test piles were driven 5 ft apart, and were embedded 18 in. in the reinforced concrete foundation blocks. No information concerning the relative density of the soil is available. It has been assumed in the analysis of the test data (Table 2) that the relative density of the sand was medium to high and that the piles were fully fixed. The lateral deflections were calculated at the maximum applied load. The calculated lateral deflections exceeded the measured ones even assuming a high relative density of the soil.

Feagin,[5] *1937*.—Feagin measured the lateral resistance of both timber and reinforced concrete piles placed by jetting in a medium dense sand. The lateral resistances of both single piles and of pile groups (containing 4, 12 and 18 piles) were investigated. The diameter of the timber piles varied from approximately 13 in. at their butt ends to approximately 9 in. at their bottom ends. The corresponding diameters for the tapered circular concrete piles were 18.0 and 10.75 in., respectively. Piles 164 BB and 152 BE were tested without end restraint, while the piles within pile groups 1, 2, 3, 5 and 6 were restrained by pile caps. Pile 164 BB and the piles within pile groups 2, 3 and 5 were timber piles while pile 152 BE was a concrete pile. Pile group 1 contained both timber and concrete piles. The ground water table was located 2 ft below the bottom of the pile caps. The calculated dimensionless length ηL exceeded 4.0. The lateral deflections of piles 164 BB and 152 BE have been calculated from Eq. 2a and the deflections of pile groups 1, 2, 3, 5 and 6 have been calculated from Eq. 2b, assuming that stiffness of the piles is governed by the diameter at the butt ends, that the piles were fully fixed at the ground surface, that the modulus of elasticity of concrete was 3×10^6 psi and that the ground water table was located at the ground surface. The measured modulus of elasticity of the material in the timber piles was 1.87×10^6 psi. The calculated lateral deflections exceeded the measured ones except for pile groups 6 and 1 (Table 2).

Shilts, Graves, and Driscoll,[31] *1948*.—The lateral resistances of poles were measured by Shilts, Graves and Driscoll. Poles 1 through 6 were placed in augered holes backfilled with compacted material. The supporting soil consisted of a layer of gravelly sand down to a depth of 4 ft. This layer was underlaid by a deep layer of poorly graded gravel of low density. No information concerning the relative density of the soil is available. The lateral deflection of the piles has been calculated at the ground surface by Eq. 2a, assuming that the poles were stiff and that the relative density of the soil was medium to low.

Laboratory tests were carried out on square model steel poles with a side of 3.0 in. and on model wooden poles with a square or circular cross-section with a side or diameter of 3.0 in. The poles were embedded in a compacted fine sand with an effective grain-size of 0.18 mm and a coefficient of uniformity of 1.63. The lateral deflections of the test poles have been calculated assuming that the relative density of the soil was medium to loose and that the poles were infinitely rigid. The lateral deflections increased approximately linearly with the applied load up to approximately half the maximum applied load. The lateral deflections have been calculated at this load level. The measured lateral deflections correspond closely to those of a sand with a medium relative density.

Loos and Breth,[14] *1949*.—Lateral deflections as well as the distribution of bending moments were measured by Loos and Breth for model piles with a diameter of 2.44 in. which were embedded in a dry compacted fine sand. Test 7 was, however, carried out at a low relative density.

The lateral deflections have been calculated from Eq. 2a assuming that the relative density of the soil corresponds to that of a loose to a medium dense sand. The reported stiffness EI of the test piles was 6.78×10^6 lb-in.^{2} The measured lateral deflections correspond closely to those calculated for a dense sand. The distribution of bending moments was also measured. The maximum bending moments have been calculated by Eq. 11 assuming that the angle of internal friction of the soil was 35° and that the unit weight was 125 lb per cu ft. Tests 7 and 8 were carried out in a sand with a low and high relative density, respectively. The corresponding angles of internal friction have been taken as 30° and 40°. The measured maximum bending moments agreed closely with the calculated moments as can be seen from Table 4.

Ramelot and Vandeperre,[23] *1950*.—A series of laboratory tests were carried out on laterally loaded wood and steel poles driven into or embedded in a sand with an angle of repose of 33.7° and a unit weight of 93.8 lb per cu ft. These values have been used in the calculation by Eq. 8 of the ultimate lateral resistance. The cross-sectional areas of the square or rectangular piles 1, 2, 3 and 4 were 3.94 in. by 3.94 in., 11.8 in. by 3.94 in., 1.97 in. by 1.97 in., and 3.94 in. by 3.94 in., respectively. The cross-sectional area of pile 5 was circular with a diameter of 3.94 in. The over-turning moment was applied to the test piles by means of vertical loads acting on a cantilever arm with a length of 39.4 in., 78.8 in., 39.4 in., 78.8 in. and 39.4 in. for piles 1, 2, 3, 4 and 5, respectively. A comparison between measured and calculated values is made in Table 3. It can be seen that the measured ultimate lateral resistance exceeded considerably the calculated values.

Raymondi,[24] *1950*.—The lateral resistances of hollow cylindrical reinforced concrete piles driven through 1.5 ft of peat into a dense sand were investigated. The outside and the inside diameters of the test piles were 13.0 and 5.9 in., respectively. The vertical piles were unrestrained and were loaded laterally at the ground surface. The lateral deflections have been calculated by Eq. 2a assuming that the piles were uncracked, that the modulus of elasticity of the pile material was 3×10^6 psi and that the relative density of the sand was medium to high. The shallow peat layer has been neglected in the calculations. Good agreement was found between measured and calculated lateral deflections (Table 2).

Knop,[11] *1952*.—Knop measured the lateral deflections of a pipe pile with an outside diameter of 20.0 in. driven through a 12-ft layer of clayey sand overlaying 6 ft of gravelly sand. The test site was covered by approximately 6 ft of water. The total penetration depth of the test pile was 16.0 ft. The lateral deflections at the ground surface have been calculated from Eq. 3a, assuming that the pile was infinitely stiff and that the relative density of the soil corresponds to that of a loose or medium dense sand. Good agreement was found between the measured and the calculated lateral deflections (Table 2).

Feagin,[7] *1953*.—Feagin measured the lateral resistance of pile groups consisting of eight vertical timber piles spaced approximately 3 ft apart. The test piles (with a diameter varying between 12 in. to 14 in. at the butt ends) were driven 30 ft through a coarse sand containing occasional gravel. The driving resistance of the test piles indicates that the relative density of the sand at the test site was low. The butt ends of the test piles were embedded 2 ft in a massive concrete block. The dimensionless length ηL exceeded 4.0. The lateral deflections of the pile groups have been calculated assuming that the ground water table was located below the significant depth (approximately 10 ft below the ground surface), that the average pile diameter at the ground surface was 13 in., that the modulus of elasticity for pile material was 1.5×10^6 psi, and that the piles were fully restrained. Good agreement was found between measured and calculated lateral deflections as can be seen from Table 2. The calculated lateral deflections are, however, affected by the assumed stiffness of the pile section EI and by the assumed value of the coefficient n_h. An increase of the pile stiffness EI or of the coefficient n_h by a factor of two will decrease the calculated lateral deflections by 24% and 35%, respectively.

O'Halloran,[20] *1953*.—O'Halloran measured the lateral resistance of a pile group consisting of eight timber piles driven into a hydraulically placed sand fill. The piles were restrained by a rigid pile cap. The average diameter of the test piles were 15.3 in. and 12.5 in. at the butt and the tip ends, respectively. In the analysis of the test data, it has been assumed that the coefficient n_h was 7 and 21 tons per cu ft, that the average pile diameter was 13.9 in., that the piles were fully restrained by the pile cap and that modulus of elasticity of the pile material was 1.5×10^6 psi. The dimensionless length ηL is equal to 4.97. The lateral deflections have been calculated from Eq. 2b (Table 2) and exceeded the measured deflections by approximately 50% at a load equal to one-half the calculated ultimate lateral resistance.

The ultimate lateral resistance of the pile group has been calculated, assuming that two plastic hinges develop at failure along each pile (Fig. 11), that the ultimate tensile strength of the pile material is 5,000 psi, that the shape factor for the pile section is 1.5, that the angle of internal friction and unit weight of the hydraulically placed sand fill is 35° and 125 lb per cu ft, respectively, and that the ground water table is located below the lower plastic hinge. The measured ultimate lateral resistance was found to exceed the calculated resistance by 51% as can be seen from Table 3.

Wagner,[34] *1953*.—The lateral resistance of two timber piles driven into a loose fine sand with an average standard penetration resistance of 5 blow per ft was measured by Wagner. Test pile 15 with butt and tip diameters of 12.0 in. and 8.0 in., respectively, was driven to a depth of 28 ft below the ground surface. Test pile 25 with butt and tip diameters of 14 in. and 10 in., respectively was driven to a depth of 32 ft. The ground water table was located close to the ground surface. The dimensionless length of embedment ηL of both test piles exceeded 4.0. The lateral deflections at the ground surface have been calculated from Eq. 2a assuming that the diameter at the butt ends governed the pile stiffness and that the modulus of elasticity of the pile material was 1.5×10^6. The calculated lateral deflections exceeded the measured lateral deflections, even assuming that the coefficient of horizontal subgrade reaction was 14.0 tons per cu ft.

Gleser,[8] *1953*.—The lateral load test reported by Gleser was carried out close to the test site for the lateral load tests reported by Feagin.[7] The diameters of the butt and tip ends of the single unrestrained timber pile were 13 in. and 10 in., respectively. The test pile was driven to a depth of 30 ft below the ground surface to a resistance of 48 tons as calculated by the Engineering News formula. The penetration resistance of the pile indicates that the relative density of the soil was low. The dimensionless length ηL exceeded 4.0. The lateral deflections of the test pile have been calculated from Eq. 2a assuming that the average diameter of the test pile was 13 in., that the modulus of elasticity of the pile material was 1.5×10^6 psi and that the coefficient of horizontal subgrade reaction was 7 and 21 tons per cu ft. At the maximum applied load, the calculated lateral deflections at the ground surface exceeded the measured deflections (Table 2).

Mason and Bishop,[15] *1953*.—Mason and Bishop measured the lateral resistance of a 44.8 ft long steel pile. The test pile with a reported stiffness EI of 2.3×10^9 lb-in.^{2}, was embedded in a sand fill. The fill was constructed by compacting sand in 6-in. layers to a density of 98 lb per cu ft. The angle of internal friction as determined from triaxial tests was 35°. The dimensionless length ηL exceeded 4.0. The lateral deflections have been calculated by Eq. 2a assuming that the coefficient of lateral subgrade reaction was 7 or 21 tons per cu ft. At the maximum applied load, the calculated lateral deflection exceeded the maximum measured lateral deflection as can be seen from Table 2.

Tschebotarioff,[32] *1953*.—Tschebotarioff carried out a series of lateral load tests on model piles driven through 14 in. of loose sand (with a relative density less than 20%) into soft clay with an unconfined compressive strength of approximately 300 lb per sq ft. The diameters of the model piles were 2.0 in. and 0.875 in. at the butt and at the tip ends, respectively. In the analysis of the test data, the average diameter of the test piles have been taken as 1.50 in., the submerged unit weight of the sand as 57 lb per cu ft, the angle of internal friction of the sand as 30° and the cohesive strength c_u of the clay as half its unconfined compressive strength. The unit lateral earth pressure for the portion of the pile embedded in the sand has been taken as three times the passive Rankine lateral earth pressure and the unit lateral earth pressure of the portion of the pile embedded in clay as 9.0 c_u. The calculated ultimate lateral resistance has been compared in Table 3 with the measured lateral resistance. Failure took place when the applied load approached twice the calculated ultimate lateral resistance.

Wen,[36] *1955.*—Wen investigated the lateral deflections of pile bents consisting of two and three vertical restrained wooden model piles with square cross-sectional areas (1-1/2 in. by 1-1/2 in.). The piles were driven 39 in. into a uniform dry sand. Dynamic penetration tests carried out by the investigator suggest that the relative density of the sand was low to medium. The lateral deflections of the piles have been calculated from Eq. 2b, assuming that the piles in each pile bent were fixed, that the modulus of elasticity of the pile material was 1.5×10^6 psi and that the relative density of the sand was low to medium. Satisfactory agreement was found between measured and calculated lateral deflection values (Table 2).

The distribution of bending moments in the laterally loaded pile bents was measured by strain gages attached to the surface of the piles. The sum of the maximum positive and negative bending moments has been calculated by Eq. 18. It has been assumed in the analysis that the angle of internal friction and the unit weight of the soil were 35° and 125 lb per cu ft, respectively. The calculated maximum bending moments agree closely with the calculated values (Table 4).

Walsenko,[35] *1955.*—The ultimate lateral resistances of poles with a diameter of 3.0 in. were measured. Tests 1, 2 and 3 were carried out in a fine sand with an angle of internal friction of 36.9° and a cohesion of 160 lb per sq ft as measured by direct shear tests. The unit weight of the sand was 123.5 lb per cu ft. The ultimate lateral resistance has been calculated assuming that the passive lateral pressure acting on the front face of the laterally loaded pile was three times the Rankine passive lateral pressure (Fig. 5). Test 3 was not carried out to failure. A comparison between the measured and calculated ultimate resistances is made in Table 3. It can be seen that the measured ultimate lateral resistances are more than twice the calculated ultimate resistances.

Tests 4 and 5 were carried out in a fine cohesionless gravel with an angle of internal friction of 45° and a unit weight of 106.0 lb per cu ft. The ultimate lateral resistance has been calculated by Eq. 8. The calculated ultimate lateral resistances were approximately two-thirds the measured values.

McNulty,[18] *1956.*—The lateral deflections of three standard Raymond cast-in-place concrete piles were measured. Test piles 1, 2 and 3 with diameters at the butt end of 18.5 in., 16.9 in. and 13.6 in., respectively, were fully free and were loaded close to the ground surface. The test piles were driven through a 2 ft to 3 ft layer of soft brown clay with an average standard penetration resistance of four blows per ft. The piles were driven into a deep stratum of a silty sand with an average standard penetration resistance of 12 blows per ft. The dimensionless length ηL was larger than 4.0. The lateral deflections of the test piles have been calculated by Eq. 2a at a load level equal to half the maximum applied lateral load, assuming that the modulus of elasticity of the pile material was 3×10^6 psi and that the relative density of the supporting sand was medium to high. A comparison between measured and calculated lateral deflections is made in Table 2. The calculated deflections exceeded the measured values.

Roscoe and Schofield,[26] *1956.*—The lateral resistance of short cast-in-place concrete piers embedded in a medium to fine sand was measured. The piers with a square cross-sectional area (18.0 × 18.0 in.) were restrained at the ground surface by tie rods which caused the piers to rotate around a point located at the ground surface. The measured average relative density of the sand was 0.3 and the measured average angle of internal friction was 39.5° The maximum moment has been calculated at the ground surface by Eqs. 16 and 18, assuming that the lateral earth pressure at failure is equal to three times the passive Rankine pressure. A comparison between measured and calculated ultimate moment (Table 4) shows good agreement.

Roscoe,[28] *1957.*—Roscoe measured the ultimate lateral resistance of model piles embedded in a dry uniform sand with a unit weight of 95 lb per cu ft and an angle of internal friction of 37°. The cross-sectional areas of piles 1 and 3 were square while the cross-sectional area of pile 2 was circular. The side and the diameter of piles 1 and 2 were 2.0 in., while the side of pile 3 was 1.41 in. The ultimate lateral resistances have been calculated by Eq. 8 and are compared with the measured values in Table 3.

California Division of Highways,[3] *1958.*—The lateral deflections of cylindrical cast-in-place reinforced concrete piles were measured. The diameter of piles I16, I16C, III16, and III16C was 16.0 in. while the diameter of piles I36N, I36S, III36N, III36S, III36CN and III36CS was 36.0 in. Piles I16C, III16C, III36CN and III36CS were restrained by a pile cap while the remainder of the test piles were free. The test site was underlaid by 10 ft of a fine to coarse sand with an average standard penetration resistance of 10 blows per ft. The lateral deflections of piles I16, I36N and I36S have been calculated by Eq. 2a and the lateral deflections of test pile I16C have been calculated by Eq. 2b. The stiffness of the pile section has been calculated on the basis of the uncracked section assuming that the modulus of elasticity of the pile material was 3×10^6 psi. Satisfactory agreement was found between measured and calculated values of lateral deflection (Table 2).

The distribution of bending moments was measured by strain gages attached to the reinforcement. The maximum bending moment for the unrestrained piles has been calculated by Eqs. 10 and 11 and the sum of the maximum negative and positive bending moments of the restrained piles has been calculated by Eq. 18, assuming that the angle of internal friction of the sand was 35° and its unit weight was 125 lb per cu ft. The calculated moments with the measured values (Table 4) agreed closely.

Loos and Breth,[14] *1959.*—The lateral resistance of a model pile with an outside diameter of 2.44 in. was measured. The test pile was embedded in a dry fine sand of a low relative density. The ultimate lateral resistance has been calculated from Eq. 8, assuming that the angle of internal friction of the soil was 30°, and that its unit weight was 125 lb per cu ft. Good agreement was found between the measured and the calculated values.

Broms,[2] *1960.*—Broms investigated the lateral resistance of a cast-in-place reinforced concrete pile driven into a loose silty sand. The average standard penetration resistance of the soil was 7 blows per ft (within the significant depth). The diameter of the pile at the ground surface and the embedment length were 15.38 in. and 53.5 ft, respectively. The dimensionless length ηL exceeded 4.0. The test pile was tested under both unrestrained and restrained conditions and the corresponding lateral deflections at the ground surface have been calculated from Eqs. 2a and 2b, respectively. The stiffness of the pile has been calculated on the basis of the uncracked cross-section neglecting the reinforcement and assuming a modulus of elasticity of the pile material of 3×10^6 psi. The lateral deflections of the unrestrained pile was, at the same lateral load, approximately three times the lateral deflections of the restrained pile. Good agreement was found between the measured and the calculated lateral deflections (Table 2). The test pile was reinforced with six No. 6 bars down to a depth of 15.0 ft below the ground surface. The average yield point of the reinforcement was 45,000 psi. The calculated yield resistance of the pile section was 54.5 kip-ft neglecting the bending resistance of the pile shell. Assuming that the angle of internal friction and the unit weight of the soil were 35° and 125 lb per cu ft, respectively, the calculated ultimate lateral resistance of the test pile was 18.06 kips as determined by Eq. 12. A comparison between the measured and the calculated ultimate lateral resistance is made in Table 3. Failure of the long laterally loaded pile took place when the applied load reached 1.77 times the calculated ultimate lateral resistance. The discrepancy between measured and calculated ultimate lateral resistance can at least partly be attributed to the strength of the corrugated pile shell, which has been neglected in the analysis.

Prakash,[21] *1962.*—The test piles used by Prakash were embedded in a medium to dense sand compacted in 6 in. layers with the aid of a vibrator. The relative density D_r of the sand as determined by a method proposed by Kolbuzevsky was 91% except for tests 13(D_r = 46.2%), 16 (D_r = 77.5%) 19 and 24 (D_r = 85.0%). The effective grain size D_{10} of the sand was 0.21 mm and its coefficient of uniformity was 1.27. The angles of internal friction as determined by triaxial drained tests were 46.6, 41.1 and 38.5 degrees for the relative densities of 90%, 65% and 30%, respectively. The diameter of the aluminum test piles was 0.50 in., and their stiffness EI was 1.4×10^3 lb-in^2

All piles were restrained by pile caps except for piles 1 and 19, which were fully free. Test 1 was carried out on a single pile; tests 3, 17, 4, 5, 6, 7, 13, 16, 19, 8, 9, 10 and 11 were carried out on a 2×2 pile group; test 24 was carried out on a 1×3 pile group; and test 21, 22 and 23 were carried out on a 3×3 pile group. The dimensionless length of the test piles exceeded 4.0. The lateral deflections of the pile groups have been determined from Eqs. 2a or 2b, assuming that the relative density of the soil corresponds to that of a medium to dense sand. The lateral deflections of test 13 (D_r = 46.2%) corresponds to that of a pile embedded in a soil of medium relative density, while the remainder of the test data (D_r = 77.5 to 91%) corresponds to those of a dense sand. Good agreement was found between measured and calculated lateral deflections (Table 2).

The distribution of lateral bending moments was measured by means of strain gages. The maximum bending moment of the free-headed piles has been calculated by Eq. 11 and the sum of the maximum positive and negative bending moments for the restrained piles has been calculated by Eq. 18. It has been assumed in the calculations that the angle of internal friction of the sand was 45° except for tests 13 and 16 when an angle of 40° was used. The unit weight of the soil has been taken as 125 lb per cu ft. A comparison between measured and calculated maximum bending moments is made in Table 4.

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DISCUSSION