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LATERAL RESISTANCE OF PILES IN COHESIVE SOILS

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SYNOPSIS

Methods are presented for the calculation of the ultimate lateral resistance and lateral deflections at working loads of single piles and pile groups driven into saturated cohesive soils. Both free and fixed headed piles have been considered. The ultimate lateral resistance has been calculated assuming that failure takes place either when one or two plastic hinges form along each individual pile or when the lateral resistance of the supporting soil is exceeded along the total length of the laterally loaded pile. Lateral deflections at working loads have been calculated using the concept of subgrade reaction taking into account edge effects both at the ground surface and at the bottom of each individual pile.

The results from the proposed design methods have been compared with available test data. Satisfactory agreement has been found between measured and calculated ultimate lateral resistance and between calculated and measured deflections at working loads. For design purposes, the proposed analyses should be used with caution due to the limited amounts of test data.

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INTRODUCTION

Single piles and pile groups are frequently subjected to high lateral forces. These forces may be caused by earthquakes, by wave or wind forces or by lateral earth pressures. For example, structures constructed off-shore, in the Gulf of Mexico, the Atlantic or Pacific Oceans, are subjected to the lateral forces caused by waves and wind.[2] The safety of these structures depends on the ability of the supporting piles to resist the resulting lateral forces.

Structures built in such areas as the states of California, Oregon, and Washington, or in Japan, may be subjected to high lateral accelerations caused by earthquakes and the supporting piles are called upon to resist the resulting lateral forces. For example, the building codes governing the design of structures in these areas specify frequently that the piles supporting such structures should have the ability to resist a lateral force equal to 10% of the applied axial load.[3,4]

Pile supported retaining walls, abutments or lock structures frequently resist high lateral forces. These lateral forces may be caused by lateral earth pressures acting on retaining walls or rigid frame bridges, by differential fluid pressures acting on lock structures or by horizontal thrust loads acting on abutments of fixed or hinged arch bridges.

The lateral bearing capacity of vertical piles driven into cohesive and cohesionless soils will be investigated in two papers. This paper is the first in that series and is concerned with the lateral resistance of piles driven into cohesive soils. Methods will be presented for the calculation of lateral deflections, ultimate lateral resistances and maximum bending moments in that order.

In the analyses developed herein, the following precepts have been assumed: (a) the deflections at working loads of a laterally loaded pile should not be so excessive as to impair the proper function of the member and that (b) its ultimate strength should be sufficiently high as to guard against complete collapse even under the most unfortunate combination of factors. Therefore, emphasis has been placed on behavior at working loads and at failure (collapse).

The behavior at working loads has been analyzed by elastic theory assuming that the laterally loaded pile behaves as an ideal elastic member and that the supporting soil behaves as an ideal elastic material. The validity of these assumptions can only be established by a comparison with test data. The behavior at failure (collapse) has been analyzed assuming that the ultimate strength of the pile section or the ultimate strength of the supporting soil has been exceeded.

It should be noted that the methods developed in this paper to predict behavior at working loads are not applicable when local yielding of the soil or of the pile material takes place (when the applied load exceeds about half the ultimate strength of the loaded member).

2 Wiegel, R. L., Beebe, K. E., and Moon, J., "Ocean-Wave Forces on Circular Cylindrical Piles," Transactions, ASCE, Vol. 124, 1959, pp. 89-113.

3 "Recommended Lateral Force Requirements," Seismology Committee, Structural Engrs. Assoc., San Francisco, Calif., July, 1959.

4 "Uniform Building Code," Pacific Coast Bldg. Officials Conf., Los Angeles, Calif., 1960.

BEHAVIOR OF LATERALLY LOADED PILES

A large number of lateral load tests have been carried out on piles driven into cohesive soils.[5-27]

5 Bergfelt, A., "The Axial and Lateral Load Bearing Capacity, and Failure by Buckling of Piles in Soft Clay," Proceedings, Fourth Internatl. Conf. on Soil Mechanics and Foundation Engrg., Vol. II, London, England 1957, pp. 8-13.

6 Browne, W. H., "Tests of North Carolina Poles for Electrical Distribution Lines," North Carolina State College of Agriculture and Engineering Experiment Station Bulletin, No. 3, Raleigh, North Carolina August, 1929.

7 Evans, L. T., "Bearing Piles Subjected to Horizontal Loads," Symposium on Lateral Load Tests on Piles, ASTM Special Technical Publication, No. 154, 1953, pp. 30-35.

8 Gaul, R. D., "Model Study of a Dynamically Laterally Loaded Pile," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 84, No. SM1, Proc. Paper 1535, February, 1958.

9 Krynine, D. P., "Land Slides and Pile Action," Engineering News Record, Vol. 107, November, 1931, p. 860.

10 Lazard, A., "Moment limite de renversement de fondations cylindriques et parallélé-pipédiques isolées," Annales de l'Institute Technique du Bâtiment et des Travaux Publics, January, Paris, France 1955, pp. 82-110.

11 Lazard, A., "Discussion," Annales de l'Institute Technique du Bâtiment et des Travaux Publics, July-August, Paris, France 1955, pp. 786-788.

12 Lazard, A., and Gallerand, G., "Shallow Foundations," Proceedings, Fifth Internatl. Conf. on Soil Mechanics and Foundation Engrg., Vol. III, Paris, France 1961, pp. 228-232.

13 Lorenz, H., "Zur Tragfähigkeit starrer Spundwände und Mastgründungen," Bautechnik-Archiv, Heft 8, Berlin, Germany 1952, pp. 79-82.

14 Matlock, H., and Ripperger, E. A., "Procedures and Instrumentation for Tests on a Laterally Loaded Pile," Proceedings, Eighth Texas Conf. on Soil Mechanics and Foundation Engrg. Research, Univ. of Texas, Austin, Tex., 1956.

15 Matlock, H., and Ripperger, E. A., "Measurement of Soil Pressure on a Laterally Loaded Pile," Proceedings, Amer. Soc. of Testing Materials, Vol. 58, 1958, pp. 1245-1259.

16 McCammon, G. A., and Ascherman, J. C., "Resistance of Long Hollow Piles to Applied Lateral Loads," Symposium on Lateral Load Tests on Piles, ASTM Special Technical Publication, No. 154, 1953, pp. 3-9.

17 McNulty, J. F., "Thrust Loadings on Piles," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 82, No. SM2, Proc. Paper 940, April, 1956.

18 Osterberg, J. O., "Lateral Stability of Poles Embedded in a Clay Soil," Northwestern University Project 208, Bell Telephone Labs., Evanston, Ill., December, 1958.

19 Parrack, A. L., "An Investigation of Lateral Loads on a Test Pile," Texas A & M Research Foundation, Research Foundation Project No. 31, College Station, Texas August, 1952.

20 Peck, R. B., and Ireland, H. O., "Full-Scale Lateral Load Test of a Retaining Wall Foundation," Fifth Internatl. Conf. on Soil Mechanics and Foundation Engrg., Paris, France, Vol. II, 1961, pp. 453-458.

21 Peck, R. B., and Davisson, M. T., discussion of "Design and Stability Considerations for Unique Pier," by James Michalos and David P. Billington, Transactions, ASCE, Vol. 127, Part IV, 1962, pp. 413-424.

22 Sandeman, J. W., "Experiments on the Resistance to Horizontal Stress of Timber Piling," von Nostrand's Engineering Magazine, Vol. XXIII, 1880, pp. 493-497.

23 Seiler, J. E., "Effect of Depth of Embedment on Pole Stability," Wood Preserving News, Vol. 10, No. 11, 1932, pp. 152-161, 167-168.

24 Shilts, W. L., Graves, L. D., and Driscoll, C. G., "A Report of Field and Laboratory Tests on the Stability of Posts Against Lateral Loads," Proceedings, Second Internatl. Conf. on Soil Mechanics and Foundation Engrg., Rotterdam, Holland Vol. V, 1948, p. 107.

25 Terzaghi, K., "Theoretical Soil Mechanics," John Wiley & Sons, Inc., New York, N. Y., 1943.

In many cases the available data are difficult to interpret. Frequently, load tests have been carried out for the purpose of proving to the satisfaction of the owner or the design engineer that the load carrying capacity of a pile or a pile group is sufficiently large to resist a prescribed lateral design load under a specific condition. In general, sufficient data are not available concerning the strength and deformation properties, the average relative density and the angle of internal friction of cohesionless soils or the average unconfined compressive strength of the cohesive soil. It is hoped that this paper will stimulate the collection of additional test data.

The load-deflection relationships of laterally loaded piles driven into cohesive soils is similar to the stress-strain relationships as obtained from consolidated-undrained tests.[28] At loads less than one-half to one-third the ultimate lateral resistance of the pile, the deflections increase approximately linearly with the applied load. At higher load levels, the load-deflection relationships become non-linear and the maximum resistance is in general reached when the deflection at the ground surface is approximately equal to 20% of the diameter or side of the pile.

The ultimate lateral resistance of a pile is governed by either the yield strength of the pile section or by the ultimate lateral resistance of the supporting soil. It will be assumed that failure takes place by transforming the pile into a mechanism through the formation of plastic hinges. Thus the same principles will be used for the analysis of a laterally loaded pile as for a statically indeterminate member or structure and it will be assumed that the moment at a plastic hinge remains constant once a hinge forms. (A plastic hinge can be compared to an ordinary hinge with a constant friction.)

The possible modes of failure of laterally loaded piles are illustrated in Figs. 1 and 2 for free headed and restrained piles, respectively. An unrestrained pile, which is free to rotate around its top end, is defined herein as a free-headed pile.

Failure of a free-headed pile (Fig. 1) takes place when (a) the maximum bending moment in the pile exceeds the moment causing yielding or failure of the pile section, or (b) the resulting lateral earth pressures exceed the lateral resistance of the supporting soil along the full length of the pile and it rotates as a unit, around a point located at some distance below the ground surface [Fig. 1(b)]. Consequently, the mode of failure depends on the pile length, on the stiffness of the pile section, and on the load-deformation characteristics of the soil. Failure caused by the formation of a plastic hinge at the section of maximum bending moment [Fig. 1(a)] takes place when the pile penetration is relatively large. Failure caused by exceeding the bearing capacity of the surrounding supporting soil [Fig. 1(b)] takes place when the length of the pile and its penetration depth are small.

The failure modes of restrained piles are illustrated in Fig. 2. Fixed-headed piles may be restrained by a pile cap or by a bracing system, as is frequently the case for bridge piers or for off-shore structures. In the case

[26] Wagner, A. A., "Lateral Load Tests on Piles for Design Information," Symposium on Lateral Load Tests on Piles, ASTM Special Publication, No. 154, 1953, pp. 59-72.

[27] Walsenko, A., "Overturning Properties of Short Piles," thesis presented to the University of Utah, at Salt Lake, Utah, in 1958, in partial fulfilment of the requirements for the degree of Master of Science.

[28] McClelland, B., and Focht, J. A., Jr., "Soil Modulus for Laterally Loaded Piles," Transactions, ASCE, Vol. 123, 1958, pp. 1049-1063.

when the length of the piles and the penetration depths are large, failure may take place when two plastic hinges form at the locations of the maximum positive and maximum negative bending moments. The maximum positive moment is located at some depth below the ground surface, while the maximum negative moment is located at the level of the restraint (at the bottom of a pile cap or at the level of the lower bracing system for pile bends).

For truly fixed-headed conditions, the maximum negative moment is larger than the maximum positive moment and hence, the yield strength of the pile section is generally exceeded first at the top of the pile. However, the pile is still able to resist additional lateral loads after formation of the first plastic hinge and failure does not take place until a second plastic hinge forms at the point of maximum positive moment. The second hinge forms when the magnitude of this moment is equal to the moment causing yielding of the pile section [Fig. 2(a)]

Failure may also take place after the formation of the first plastic hinge at the top end of the pile if the lateral soil reactions exceed the bearing capacity of the soil along the full length of the pile as shown in Fig. 2(b), and the pile rotates around a point located at some depth below the ground surface. The mode of failure, shown in Fig. 2(b), takes place at intermediate pile lengths and intermediate penetration depths. When the lengths of the piles and the penetration depths are small, failure takes place when the applied lateral load exceeds the resistance of the supporting soils, as shown in Fig. 2(c). In this case, the action of a pile can be compared to that of a dowel.

Methods of computing the distribution of bending moments, deflections and soils reactions at working loads (at one-half to one-third the ultimate lateral resistance) are reviewed in the following section and a method for the calculation of the ultimate lateral resistance of free and restrained piles is presented in a succeeding section.

Notation.—The symbols adopted for use in this paper are defined where they first appear and are arranged alphabetically in Appendix I.

BEHAVIOR AT WORKING LOADS

At working loads, the deflections of a single pile or of a pile group can be considered to increase approximately linearly with the applied load. Part of the lateral deflection is caused by the shear deformation of the soil at the time of loading and part by consolidation and creep subsequent to loading. (Creep is defined as the part of the shear deformations which take place after loading.) The deformation caused by consolidation and creep increases with time.

It will be assumed in the following analysis, that the lateral deflections and the distribution of bending moments and shear forces can be calculated at working loads by means of the theory of subgrade reaction. Thus, it will be assumed that the unit soil reaction p (in pounds per square inch or tons per square foot) acting on a laterally loaded pile increases in proportion to the lateral deflection y (in inches or feet) expressed by the equation

$$p = k\,y \quad \text{. . . . .} \quad (1)$$

where the coefficient k (in pounds per cubic inch or tons per cubic foot) is defined as the coefficient of subgrade reaction. The numerical value of the

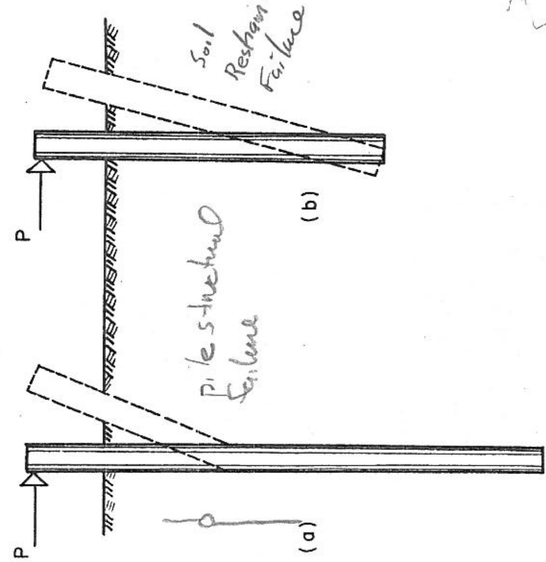

FIG. 1.—FAILURE MODES FOR FREE-HEADED PILES

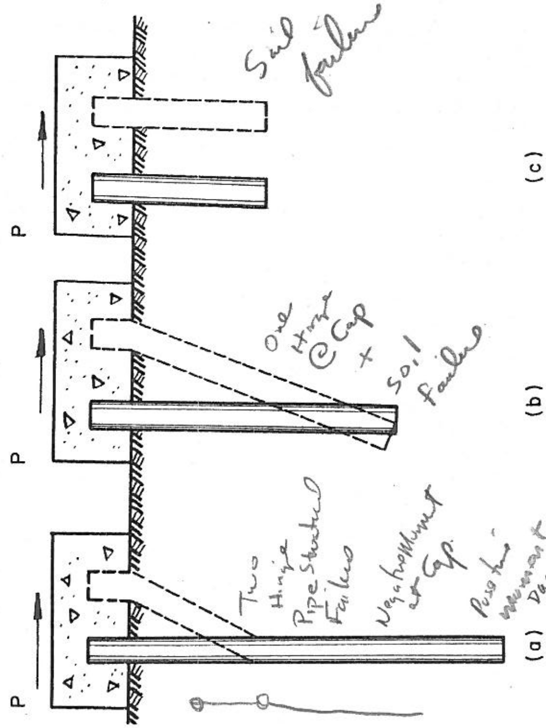

FIG. 2.—FAILURE MODES FOR RESTRAINED PILES

coefficient of subgrade reaction varies with the width of the loaded area and the load distribution, as well as with the distance from the ground surface.[29-33]

The corresponding soil reaction per unit length Q (in pounds per inch or tons per foot) can be evaluated from

$$Q = k\,D\,y \quad \ldots\ldots\ldots\ldots \quad (2)$$

in which D is the diameter or width of the laterally loaded pile. If k D is denoted K (in pounds per square inch or tons per square foot), then

$$Q = K\,y \quad \ldots\ldots\ldots\ldots \quad (3)$$

Methods for the evaluation of the coefficient K for piles driven into cohesive soils have been discussed by Terzaghi[27] and will be summarized subsequently. However, the numerical value of this coefficient is affected by consolidation and creep.

In the following analysis, it will be assumed that the coefficient of subgrade reaction is constant within the significant depth. (The significant depth is defined as the depth wherein a change of the subgrade reaction will not affect the lateral deflection at the ground surface or the maximum bending moment by more than 10%.)

However, the coefficient of subgrade reaction is seldom a constant but varies frequently as a function of depth. It will be shown that the coefficient of subgrade reaction for cohesive soils is approximately proportional to the unconfined compressive strength of the soil.[34] As the unconfined compressive strength of normally consolidated calys and silts increases approximately linear with depth, the coefficient of subgrade reaction can be expected to increase in a similar manner as indicated by field data obtained by A. L. Parrack[19] and by Ralph B. Peck, F. ASCE and M. T. Davisson.[21] The unconfined compressive strength of overconsolidated clays may be approximately constant with depth if, for example, the overconsolidation of the soil has been caused by glaciation while the unconfined compressive strength may decrease with depth if the overconsolidation has been caused by desiccation. Thus, the coefficient of subgrade reaction may, for an overconsolidated clay, be either approximately constant or decrease as a function of depth.

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29 Boit, A. M., "Bending of an Infinite Beam on an Elastic Foundation," Journal of Applied Mechanics, Vol. 4, No. 1, A1-A7, 1937.

30 DeBeer, E. E., "Computation of Beams Resting on Soil," Proceedings, Second Internatl. Conf. on Soil Mechanics and Foundation Engrg., Vol. 1, 1948, Rotterdam, Holland, pp. 119-121.

31 Terzaghi, K., "Evaluation of Coefficients of Subgrade Reaction," Géotechnique, London, England, Vol. V, 1955, pp. 297-326.

32 Vesić, A. B., "Bending of Beams Resting on Isotropic Elastic Solid," Journal of the Engineering Mechanics Division, ASCE, Vol. 87, No. EM2, Proc. Paper 2800, April, 1961, pp. 35-53.

33 Vesić, A. B., "Beams on Elastic Subgrade and the Winkler's Hypothesis," Proceedings, Fifth Internatl. Conf. on Soil Mechanics and Foundation Engrg., Vol. I, 1961, Paris, France pp. 845-850.

34 Skempton, A. W., "The Bearing Capacity of Clays," Building Research Congress, London, England 1951, pp. 180-189.

The limitations imposed on the proposed analysis by the assumption of a constant coefficient of subgrade reaction can be overcome. It can be shown that the lateral deflections can be predicted at the ground surface when the coefficient of subgrade reaction increases with depth if this coefficient is assumed to be constant and if its numerical value is taken as the average within a depth equal to 0.8 β L.

Lateral Deflections.—For the case when the coefficient of subgrade reaction is constant with depth, the distribution of lateral deflections, bending moments and soil reactions can be calculated numerically,[35,36,37] analytically,[38,39] or by means of models.[40] Solutions are also available for the case when a laterally loaded pile has been driven into a layered system consisting of an upper stiff crust and a lower layer of soft clays.[41]

The deflections, bending moments and soil reactions depend primarily on the dimensionless length β L, in which β is equal to $\sqrt[4]{kD/4E_pI_p}$.[35-39] In this expression, E_pI_p is the stiffness of the pile section, k the coefficient of a subgrade reaction, and D the diameter or width of the laterally loaded pile. A. B. Vesić,[32] M. ASCE has shown that the coefficient of subgrade reaction can be evaluated assuming that the pile length is large when the dimensionless length β L is larger than 2.25. In the case when the dimensionless length of the pile β L is less than 2.25, the coefficient of subgrade reaction depends primarily on the diameter of the test pile and on the penetration depth.[29,30,31,32,33]

It can be shown that lateral deflection y_o at the ground surface can be expressed as a function of the dimensionless quantity y_o k D L/D. This quantity is plotted in Fig. 3 as a function of the dimensionless pile length β L. The lateral deflections as shown in Fig. 3 have been calculated for the two cases when the pile is fully free or fully fixed at the ground surface. Frequently, the laterally loaded pile is only partly restrained and the lateral deflections at the ground surface will attain values between those corresponding to fully fixed or fully free conditions.

The lateral deflections at the ground surface can be calculated for a free-headed pile as can be seen from Fig. 3 assuming that the pile is infinitely stiff when the dimensionless length β L is less than 1.5. For this case, the lateral deflection is equal to:

$$y_o = \frac{4\ P\ \left(1 + 1.5\ \frac{e}{L}\right)}{k\ D\ L} \quad \ldots\ldots\ldots\ldots\ (4a)$$

35 Gleser, S. M., "Lateral Load Tests on Vertical Fixed-Head and Free-Head Piles," ASTM Special Publication, No. 154, 1953, pp. 75-93.

36 Howe, R. J., "A Numerical Method for Predicting the Behavior of Laterally Loaded Piling," Shell Oil Co., TS Memorandum 9, Houston, Tex., May, 1955.

37 Newmark, N. M., "Numerical Procedure for Computing Deflections, Moments, and Buckling Loads," Transactions, ASCE, Vol. 108, 1943, pp. 1161-1188.

38 Chang, Y. L., discussion of "Lateral Pile-Loading Tests," by Lawrence B. Feagin, Transactions, ASCE, Vol. 102, 1937, pp. 272-278.

39 Hetenyi, M., "Beams on Elastic Foundation," Univ. of Michigan Press, Ann Arbor, Mich., 1946.

40 Thoms, R. L., "A Model Analysis of a Laterally Loaded Pile," thesis presented to the University of Texas, at Austin, Tex., in 1957, in partial fulfilment of the requirements for the degree of Master of Science.

41 Davisson, M. T., and Gill, H. L., "Laterally Loaded Piles in a Layered System," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 89, No. SM3, Proc. Paper 3509, May, 1963, pp. 63-94.

A restrained pile with a dimensionless length β L less than 0.5 behaves as an infinitely stiff pile (Fig. 3) and the lateral deflection at the ground surface can be calculated directly from the equation

$$y_o = \frac{P}{k\ D\ L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ (4b)$$

It should be noted that an increase of the pile length decreases appreciably the lateral deflection at the ground surface for short piles (β L less than 1.5 and 0.5 for free-headed and restrained piles, respectively). However, a change of the pile stiffness has only a small effect on the lateral deflection for such piles. The lateral deflections at the ground surface of short fixed piles are theoretically one-fourth or less of those for the corresponding free-headed piles (Eqs. 4a and 4b).

Thus, the provision of end restraint is an effective means of decreasing the lateral deflections at the ground surface of a long laterally loaded pile. This has been shown clearly by the tests reported by G. A. McCammon, F. ASCE, and J. C. Ascherman, M. ASCE.[16] These tests indicate that the lateral deflection of a free pile driven into a soft clay deflected at the same lateral load on the average 2.6 times as much as the corresponding partially restrained pile.

The lateral deflections at the ground surface of a free-headed pile can be calculated assuming that the pile is infinitely long (Fig. 3) when the dimensionless length β L exceeds 2.5. For this case (β L larger than 2.5) the lateral deflection can be computed directly from

$$y_o = \frac{2\ P\ \beta\ (e\ \beta + 1)}{k_\infty\ D} \quad \ldots\ldots\ldots\ldots\ldots\ (5a)$$

in which k_∞ is the coefficient of subgrade reaction corresponding to an infinitely long pile.

A restrained pile behaves as an infinitely long pile when the dimensionless length β L exceeds 1.5 as can be seen from Fig. 3. The corresponding lateral deflection (β L larger than 1.5) can be calculated from

$$y_o = \frac{P\ \beta}{k_\infty\ D} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ (5b)$$

The lateral deflections at the ground surface depends on the value of the coefficient of subgrade reaction within the critical depth. This depth can be determined from the following considerations. It can be seen from Fig. 3 that the lateral deflections at the ground surface are approximately 10% larger than those calculated assuming that the pile is infinitely long when the dimensionless pile length or embedment length β L is equal to 2.0 and 1.0 for restrained and free-headed piles, respectively. Thus the properties of the piles or of the soil beyond these dimensionless depths have only a small effect on the lateral deflections at the ground surface. The dimensionless depths β L of 2.0 and 1.0 are therefore the critical depths for restrained and free-headed piles, respectively.

$\beta L = 2.0$ Critical Depth for restrained end piles

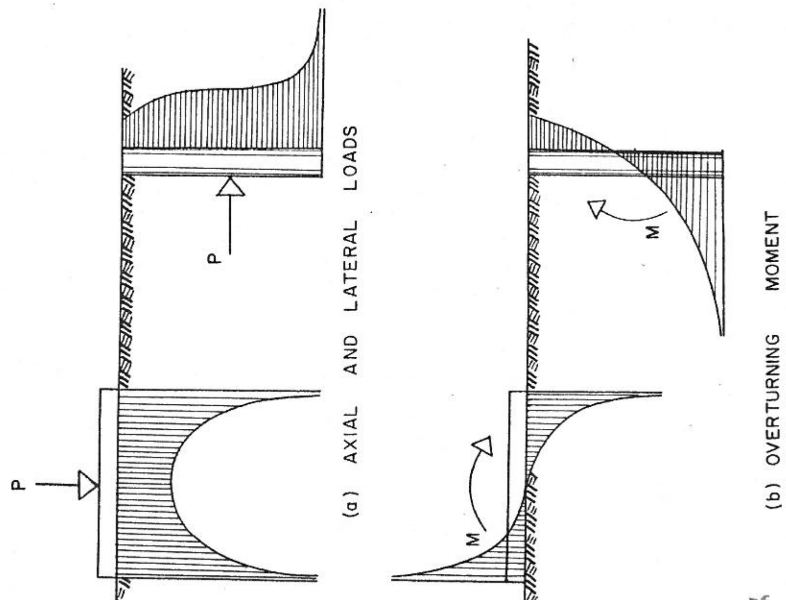


FIG. 4.—DISTRIBUTION OF SOIL REACTIONS

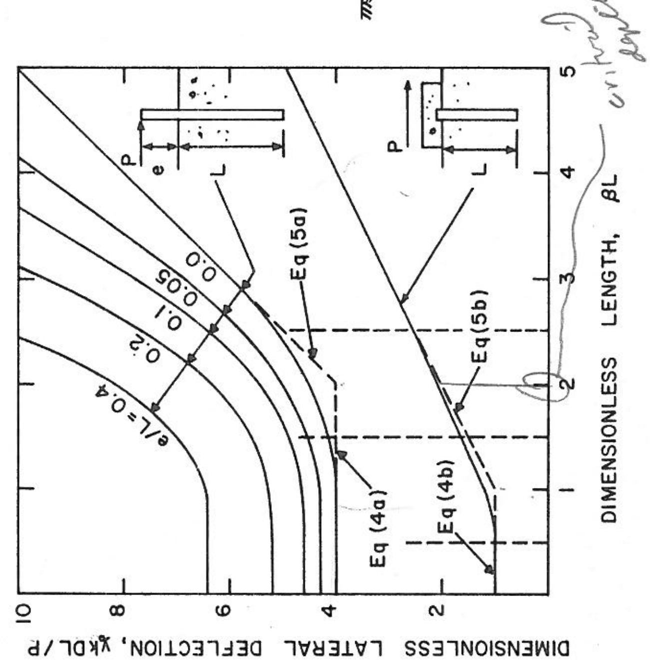


FIG. 3.—COHESIVE SOILS—LATERAL DEFLECTIONS AT GROUND SURFACE

Coefficient of Subgrade Reaction.—In the following analysis, the coefficient of subgrade reaction has been computed assuming that it is equal to that of a strip founded on the surface of a semi-infinite, ideal elastic medium. Thus, it has been assumed that the distribution of bending moments, shear forces, soil reactions, and deflections are the same for the horizontal and the vertical members shown in Fig. 4. However, the actual distribution of these quantities will be different for these two members although some of the differences tend to cancel each other. For example, due to edge effects, the coefficient of subgrade reaction at the head of the vertical member will be less than the average coefficient of subgrade reaction for the horizontal member. Furthermore, since the vertical member is surrounded on all sides by the elastic medium, the average coefficient of lateral subgrade reaction will be larger than that of the horizontal member. Thus, the deformations at the head of the laterally loaded vertical members, calculated by the following method, are only approximate and can be used only as an estimate. If it is required to determine the lateral deflections accurately, then field tests are required.

Long Piles ($\beta L > 2.25$).—Vesić[32,33] has shown that the coefficient of subgrade reaction, k, for an infinitely long strip with the width D, (such as a wall footing founded on the surface of a semi-infinite, ideal elastic body) is proportional to the factor α and the coefficient of subgrade reaction K_o for a square plate, with the length equal to unity. The coefficient k_∞ can be evaluated from

$$k_\infty = \frac{\alpha K_o}{D} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

The factor α is equal to $0.52 \sqrt[12]{\frac{K_o D^4}{E_p I_p}}$ where $E_p I_p$ is the stiffness of the loaded strip or plate. In the following analysis, it will be assumed that the coefficient of lateral subgrade reaction can be calculated from Eq. 6 and that this coefficient can be used for the determination of the distribution of bending moments, shear forces and deflections in laterally loaded piles.

Numerical calculations by the writer have indicated that the coefficient α can only vary between narrow limits for steel, concrete or timber piles. It can be determined approximately from the expression

$$\alpha = n_1 n_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

in which n_1 and n_2 are functions of the unconfined compressive strength of the supporting soil and of the pile material, respectively, as indicated in Tables 1 and 2. The coefficient α has been evaluated for steel pipe and H-piles as well as for cast-in-place or precast concrete piles with cylindrical cross sections. The minimum value of 0.29 was calculated for steel H-piles driven into a very soft clay and loaded in the direction of their largest moment resistance. The maximum value of 0.54 was calculated for timber piles driven into very stiff clays.

As an example, the factor α is equal to 0.36 (1.00×0.36) as calculated from Eq. 7 for a 50 ft long steel pipe pile driven into a clay with an unconfined compressive strength of 1.0 ton per sq ft. The corresponding coefficient of subgrade reaction is equal to 18.0 (0.36×50) tons per sq ft when the coefficient K_o is equal to 50 tons per sq ft. The coefficient K_o corresponds to the coefficient of subgrade reaction of a plate with a diameter of 1.0 ft.

The coefficient of subgrade reaction increases frequently with depth. Calculations have indicated that the lateral deflections can be calculated if it is assumed that the coefficient of subgrade reaction is a constant and that its numerical value is equal to that corresponding to the dimensionless depth βL of 0.4. For the case when the coefficient of subgrade reaction decreases with depth, the method developed by Davisson and H. L. Gill,[41] A. M. ASCE can be used.

For long piles, the calculated lateral deflections are insensitive to the assumed value of the coefficient of subgrade reaction. If, for example, the coefficient of subgrade reaction is half the assumed value, then the deflections at the ground surface will exceed the calculated deflections by about 20%. As a result, it is, in general, sufficient to estimate the magnitude of the coefficient of subgrade reaction.

TABLE 1.—EVALUATION OF THE COEFFICIENT n_1 (EQ. 7)

Unconfined Compressive Strength q_u, tons per square foot	Coefficient n_1
Less than 0.5	0.32
0.5 to 2.0	0.36
Larger than 2.0	0.40

TABLE 2.—EVALUATION OF THE COEFFICIENT n_2 (EQ. 7)

Pile Material	Coefficient n_2
Steel	1.00
Concrete	1.15
Wood	1.30

Short Piles ($\beta L < 2.25$).—The coefficient of subgrade reaction for laterally loaded short piles with a length βL less than 2.25 may be calculated approximately by the following method.

Short piles will behave under lateral load as if they are infinitely stiff and a lateral load P acting at mid-height will cause a pure translation of the pile as shown in Fig. 5(a). A moment M acting at mid-height of the pile will result in a pure rotation with respect to the center of the pile and the distribution of lateral earth pressures will be approximately triangular as shown in Fig. 5(b) (assuming a constant coefficient of subgrade reaction). It should be noticed that any force system acting on a pile can be resolved into a single lateral force and a moment acting at the center of the embedded section of the loaded

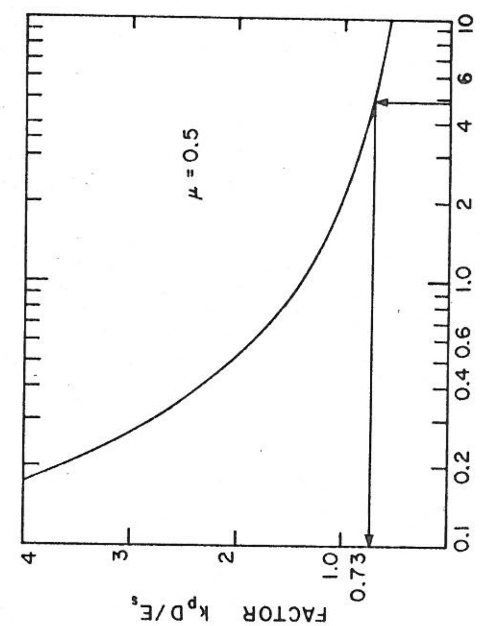


FIG. 6.—EVALUATION OF COEFFICIENT OF SUBGRADE REACTION

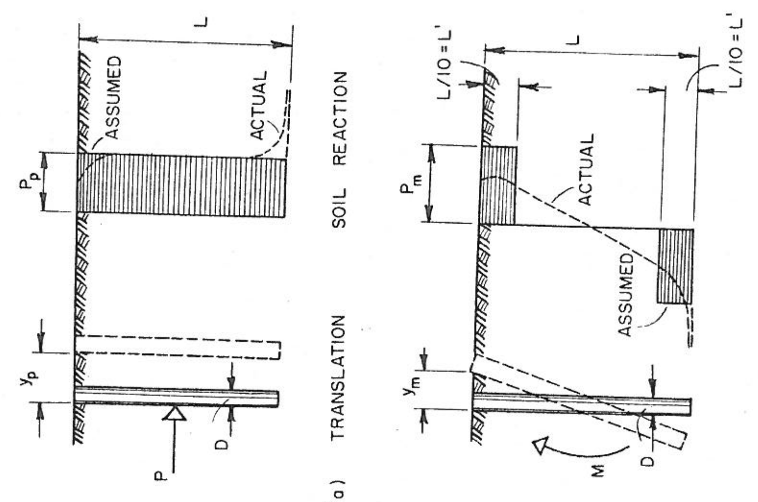


FIG. 5.—CALCULATION OF LATERAL DEFLECTIONS FOR A SHORT PILE

pile. The lateral deflection at the ground surface can be calculated by the principle of superposition with the aid of the coefficients of subgrade reaction k_p and k_m. The coefficient k_p governs the lateral deflections caused by the lateral load P whereas the coefficient k_m governs the lateral deflections caused by the moment M. The distribution of the applied load as well as the size and shape at the loaded area affect the coefficients k_p and k_m. It should be noted that numerical values of the two coefficients k_p and k_m are not the same.

The lateral deflection y_p [equal to $P/(DLk_p)$] caused by the lateral load P acting at mid-height of the laterally loaded pile, depends on the projected area LD and the coefficient of lateral soil reaction k_p. The numerical value of the coefficient k_p depends in its turn on the shape of the loaded area and can, at low load levels when the deflections are proportional to the applied load, be evaluated from theory of elasticity[42] by the equation

$$k_p = \frac{E_s}{m\left(1 - \mu_s^2\right)\sqrt{LD}} \qquad \text{(8)}$$

In Eq. 8 E_s is the modulus of elasticity of the soil, μ_s the Poisson's ratio of

TABLE 3.—NUMERICAL VALUE OF COEFFICIENT m

Ratio, L/D	1.0	1.5	2	3	5	10	100
Coefficient, m	0.95	0.94	0.92	0.88	0.82	0.71	0.37

the soil, LD the projected area of the pile and m a numerical factor which depends on the shape of the loaded area. The coefficient m is tabulated in Table 3 as a function of the ratio L/D.

The factor $k_p D/E_s$ has been plotted in Fig. 6 as a function of the ratio L/D assuming that the Poisson's ratio of the soil μ_s is equal to 0.50. It can be seen, for example, that the dimensionless quantity $k_p D/E_s$ attains a value of 0.73 at a value of L/D equal to 5.0.

The lateral deflection y_m at the ground surface caused by a moment M acting at mid-height of a laterally loaded pile has been assumed to be the same as the edge deflection of a plate located on the ground surface and loaded by the same moment M as shown in Fig. 4(b). The rotation and deflections of a stiff plate with an arbitrary shape and size located on the ground surface can be calculated by a method proposed by G. F. Weissmann and S. R. White.[43] Calculations have indicated that this method can be approximated by assuming that the soil reactions are uniformly distributed along 1/10 the total length of the member [Fig. 5(b)] and that the coefficient of subgrade reaction k_m is governed by the shape and size of the reduced area. For the specific case

[42] Timoshenko, S. and Goodier, J. N., "Theory of Elasticity," Second Edition, McGraw-Hill Book Co., Inc., New York, N. Y., 1951.

[43] Weissmann, G. F., and White, S. R., "Small Angular Deflections of Rigid Foundations," Géotechnique, London, England, Vol. XI, No. 3, September, 1961, pp. 186-202.

when the total length L is equal to the width D, the rotation and the deflection predicted by the proposed simplified method (assuming that the equivalent width of the loaded area is 1/10 the total length) are 3% smaller than those calculated by the more accurate method proposed by Weissmann and White.[43]

The deflections at the center of each of the equivalent areas is equal to p_m/k_m, in which p_m is the equivalent uniformly distributed pressure acting at top and bottom of the pile. The equivalent pressure p_m is equal to $10M/0.9\,D\,L^2$ since the internal moment arm is 0.9L and the soil reactions are distributed over 1/10 the total length of the pile. The lateral deflections at the ground surface is equal to 1/0.9 the deflections at the center of the loaded equivalent area. For these conditions, the lateral deflections y_m at the ground surface is equal to $12.35\ M/\left(DL^2k_m\right)$.

The coefficient k_m, which is the coefficient of subgrade reaction corresponding to the shape and size of the two equivalent rectangular areas, can be evaluated from Eq. 8 or directly from Fig. 6. It should be noted that the equivalent length L' (Fig. 5) is 1/10th the total length of the pile.

The coefficient of subgrade reaction increases frequently with depth. Calculations have indicated that the lateral deflections at the ground surface can be calculated assuming a constant value of this coefficient if its numerical value is taken as that corresponding to a depth of 0.25L and 0.50L for free-headed and restrained short piles, respectively.

The calculated lateral deflections are inversely proportional to the assumed value of the coefficient of subgrade reaction. If it required to predict closely the deflections of a laterally loaded short pile, then an accurate estimate of the coefficient of subgrade reaction is required.

Plate Load Tests.—The coefficient of subgrade reaction k_o and the modulus of elasticity E_s may be evaluated approximately by plate load tests with the aid of the theory of elasticity. However, it should be noted that the initial modulus of elasticity of the soil varies frequently with the direction of loading. W. H. Ward, S. G. Samuels and M. E. Butler,[44] for example, have found for the heavily overconsolidated London clay that the initial modulus of elasticity in the lateral directions exceeded the initial modulus in the vertical direction on the average by a factor of 1.6. Thus, for a heavily overconsolidated clay, the results of plate loading tests may underestimate the initial modulus of elasticity of the soil and the coefficient of lateral subgrade reaction. The initial modulus of elasticity is, for a cohesive soil, approximately proportional to its unconfined compressive strength.[34] The shearing strength of a normally consolidated clay increases in general with depth while the shearing strength of an overconsolidated clay may increase or decrease with depth. Therefore, the initial modulus of elasticity may also increase or decrease with depth.

In the analysis of plate load tests, it is, in general, assumed for cohesive soils that the modulus of elasticity of the soil is a constant. For this assumption, plate load tests will underestimate the effective lateral coefficient of subgrade reaction if the shearing strength and the modulus of elasticity increases with depth since the deflections for the plate load test depend mainly on the modulus of elasticity of the soil within a distance of approximately two plate diameters below the ground surface. On the other hand, the coef-

[44] Ward, W. H., Samuels, S. G., and Butler, M. E., "Further Studies of the Properties of London Clay," Géotechnique, London, England, Vol. IX, 1959, pp. 33-58.

ficient will be overestimated if the shearing strength and the soil modulus decrease with depth.

Remolding of the soil (as a result of pile driving) cause a decrease of the initial modulus and the secant modulus to a distance of approximately one pile diameter from the surface of the pile. Consolidation on the other hand causes a substantial increase with time of the shearing strength, of the initial and of the secant moduli for normally or lightly overconsolidated clays. However, the shearing strength and the secant modulus for heavily overconsolidated clays may decrease with time.

The deflections at working loads (approximately one-half to one-third the ultimate bearing capacity) are proportional to the secant modulus of the soil when the modulus is determined at loads corresponding to between one-half and one-third the ultimate strength of the soil.[34] This secant modulus may be considerably less than the initial tangent modulus of elasticity of the soil.[45] In the following analysis, the secant modulus E_{50} corresponding to half the ultimate strength of the soil will be assumed to govern the lateral deflections at working loads. (The assumption has been made also by A. W. Skempton[34] in the analysis of the initial deflections of spread footings founded at or close to the ground surface.) The deflection d_o of a circular plate can then be calculated from the equation[42]

$$d_o = \frac{0.8\ B\ q\left(1 - \mu_s^{\ 2}\right)}{E_{50}} \qquad (9)$$

in which B is the diameter of the loaded area, q denotes the intensity of the applied load and μ_s refers to Poisson's ratio. Since q/d_o is equal to the coefficient of subgrade reaction k_o, it can be seen that the coefficient of subgrade reaction is indirectly proportional to the diameter of B of the loaded area. If k_oB is defined as K_o and the Poisson's ratio is taken as 0.5, then

$$K_o = 1.67\ E_{50} \qquad (10)$$

Skempton[34] has found that the secant modulus E_{50} is approximately equal to 25 to 100 times the unconfined compressive strength of a cohesive soil. Analysis of test data reported by Peck and Davisson[21] on the behavior of a laterally loaded H-pile driven into a normally consolidated, highly organic silt indicates, at the maximum applied load, that the secant modulus load is approximately equal to 100 times the cohesive strength as measured by field vane tests (50 times the unconfined compressive strength of the soil).

Using a value of E_{50} equal to 25 to 100 times the unconfined compressive strength, the coefficient K_o can be expressed in terms of the unconfined compressive q_u (Eq. 10) as

$$K_o = (40 - 160)\ q_u \qquad (11)$$

45 Terzaghi, K., and Peck, R. B., "Soil Mechanics in Engineering Practice," John Wiley & Sons, Inc., New York, N. Y., 1948.

This range of values compares well with those proposed by Karl Terzaghi.[31] For clays with unconfined compressive strengths of 1.0, 2.0, and 4.0 tons per sq ft, the calculated values of the coefficients of subgrade reaction (using the relationship $K_o = 80q_u$) are 80, 160 and 320 tons/sq ft, respectively. These values compare well with the values 75, 160 and 300 tons per sq ft, proposed by Terzaghi.[31]

The coefficient of subgrade reaction K_o determined from plate load tests can also be used directly in the case of long piles ($\beta L > 2.25$) to calculate the corresponding coefficient of subgrade reaction (Eq. 6).

The secant modulus as determined from Eq. 10 can be used for calculation of the coefficient of lateral subgrade reaction of short piles ($\beta L < 2.25$). It should be noted that both for long and short piles, the coefficient of subgrade reaction has been assumed to be the same in the vertical as in the lateral directions and that it does not vary with the depth below the ground surface.

Lateral Load Tests.—Coefficients of lateral subgrade reaction can be determined also from lateral load tests on long piles with a dimensionless length β L larger than 2.25. The coefficient of lateral subgrade reaction (assuming a constant value of this coefficient within the significant depth) can then be calculated from Eqs. 4a, 4b, 5a or 5b.

Deformations Caused by Consolidation.—As a result of consolidation and creep of the soil surrounding a laterally loaded pile, an increase of the lateral deflections and a redistribution of soil reactions will occur with time. The writer is not aware of any test data concerned with the behavior of laterally loaded piles under long-time loading. The following analysis is therefore only approximate and has not been substantiated by test data. The deformations caused by consolidation depend on the nature of the applied load, on the redistribution of soil reaction along the pile, on the stress increase in the soil to a distance of approximately two to three pile diameters from the pile surface and on the compressibility of the soil.

It will be assumed that the increase of deflections of a laterally loaded pile caused by consolidation is the same as the increase of deflections (settlements) which take place with time for spread footings and rafts founded at the ground surface or at some depth below the ground surface. Test data[46] indicate that the total settlement (the sum of the initial compression and consolidation) of footings and rafts located on stiff to very stiff clays is approximately equal to two to four times the initial settlements caused by shear deformations of the soils. It is therefore reasonable to assume that the apparent coefficient of subgrade reaction for these soils which governs the long-time lateral deflections and the long-time distribution of lateral earth pressures should be taken as 1/2 to 1/4 the initial coefficient of subgrade reaction.

For normally consolidated clays, A. W. Skempton and L. Bjerrum[46] found that the total settlements are approximately three to six times the initial settlements which take place at the time of loading. The corresponding apparent coefficient of subgrade reaction governing the long-time pressure distribution of piles driven into soft and very soft clays may, therefore, be taken as 1/3 to 1/6 the initial value (Eq. 11).

46 Skempton, A. W., and Bjerrum, L., "A Contribution to the Settlement Analysis of Foundations on Clay," Géotechnique, London, England, Vol. VII, 1957, pp. 168-178.

Consolidation and creep cause an increase in lateral deflections of short piles (βL less than 1.5 and 0.5 for free-headed and restrained piles, respectively) which is inversely proportional to the decrease in the coefficient of lateral subgrade reaction as indicated by Eqs. 4a and 4b. A decrease of this coefficient, for example, to one-third its initial value will cause an increase of the initial lateral deflections at the ground surface by a factor of three. In the case of a long pile (βL larger than 2.5 and 1.5 for a free-headed and a restrained pile, respectively) the increase of lateral deflection (Eqs. 5a and 5b) caused by consolidation and creep is less than that of a short pile.

The increase in lateral deflections caused by consolidation may also be calculated by means of a settlement analysis based on the assumption that the distribution of soil reactions along the laterally loaded piles is governed by a reduced coefficient of lateral soil reaction, that the distribution of the soil pressure within the soil located in front of the laterally loaded pile can be calculated, for example, by the 2:1 method or by any other suitable method and that the compressibility of the soil can be evaluated by consolidation tests or from empirical relationships. (The 2:1 method assumes that the applied load is distributed over an area which increases in proportion to the distance to the applied load. This method closely approximates the stress distribution calculated by the theory of elasticity along the axis of loading.) Because these proposed methods of calculating lateral deflections have not been substantiated by test data they should be used with caution.

Comparison with Test Data.—The lateral deflections at working loads can be calculated by the hypothesis previously presented if the stiffness of the pile section, the pile diameter, the penetration depth, and the average unconfined compressive strength of the soil are known within the significant depth. Frequently only fragmentary data concerning the strength properties of the supporting soil are available.

The lateral deflections calculated from Eqs. 4a, 4b, 5a and 5b have been compared in Table 4 with test data reported by W. L. Shilts, F. ASCE, L. D. Graves, F. ASCE and C. G. Driscoll,[24] by Parrack,[19] by J. F. McNulty,[17] F. ASCE, by J. O. Osterberg,[18] F. ASCE, and by Peck and Davisson.[21] In the analysis of these test data, it has been assumed that the moduli of elasticity for the pile materials wood, concrete and steel are 1.5×10^6, 3×10^6 and 30×10^6 psi, respectively, and that the ratio E_{50}/q_u is equal to 50. The test data are examined in detail in Appendix II. It can be seen from Table 4 that the measured lateral deflections at the ground surface varied between 0.5 to 3.0 times the calculated deflections.

It should be noted that the calculated lateral deflections are for short piles inversely proportional to the assumed coefficient of subgrade reactions and thus to the measured average unconfined compressive strength of the supporting soil. Thus small variations of the measured average unconfined compressive strength will have large effects on the calculated lateral deflections. It should also be noted that the agreement between measured and calculated lateral deflections improves with decreasing shearing strength of the soil. The cohesive soils reported with a high unconfined compressive strength have been preloaded by desiccation and it is well known that the shearing strength of such soils is erratic and may vary appreciably within short distances due to the presence of shrinkage cracks.

The test data indicate that the proposed method can be used to calculate the lateral deflections at working loads (at load levels equal to one-half or

TABLE 4.—LATERAL DEFLECTIONS

Pile Test (1)	Pile Diameter D, in feet (2)	Eccentricity e, in feet (3)	Depth of Embedment L, in feet (4)	Applied load P, in kips (5)	Average Unconfined Compressive Strength q_u, in tons per sq ft (6)	Measured Lateral Deflection y_{test}, in inches (7)	Calculated Lateral Deflection y_{calc}, in inches (8)	Ratio y_{test}/y_{calc} (9)
Shilts, Graves and Driscoll,[24] 1948								
9[a]	1.17	9.0	6.5	2.1		0.95	0.35	2.71
10	2.0	10.0	5.0	2.0	1.53	0.40	0.34	1.18
14	2.0	10.0	6.0	3.0		0.20	0.42	0.48
Parrack,[19] 1952								
1	2.0	-[b]	75.0	20	0.37	0.098	0.107	0.92
				40		0.214	0.219	0.98
				60		0.418	0.324	1.29
				80		0.656	0.428	1.53
McNulty,[17] 1956								
A	1.0	-[b]	50.0	5	1.20[c]	0.05	0.15	0.33
				10		0.21	0.29	0.72
				15		0.50	0.44	1.13
				20		0.86	0.58	1.48
B	1.0	-[b]	50.0	5	1.20[c]	0.08	0.15	0.53
				10		0.27	0.29	1.04
				15		0.57	0.44	1.29
				20		0.95	0.58	1.63
Osterberg,[18] 1958								
T1	0.90	15.0	6.00	2.91	2.22	0.82	0.400	2.07
T2	0.90	15.0	6.00	2.42	2.22	1.25	0.333	3.75
T3	0.90	15.0	4.00	1.42	2.22	0.71	0.324	2.19
T4	0.90	15.0	4.00	1.42	2.22	0.71	0.324	2.19
T5	0.90	15.0	7.79	4.90	2.22	0.83	0.501	1.66
T6	0.90	15.0	9.50	3.91	2.64	0.42	0.272	1.55
T7	0.90	15.0	5.84	2.91	2.37	1.07	0.385	2.97
T8	1.50	15.0	6.00	4.87	1.84	0.53	0.601	0.88
T9	2.00	15.0	6.00	5.81	2.40	0.25	0.466	0.54
T10	2.00	15.0	6.00	4.87	3.03	0.37	0.311	1.19
T11	2.67	14.0	6.00	9.78	3.05	0.32	0.489	0.65
T12	3.09	13.2	6.00	12.75	3.05	0.49	0.471	1.07
T13	1.50	15.0	6.00	6.27	2.25	0.77	0.535	1.44
T14	0.90	15.0	6.16	3.41	3.05	0.94	0.326	3.09
Peck and Davisson,[21] 1962								
1	H-pile (14BD89)	32.5	54.3	1.0	0.400[d]	0.6	0.63	0.95
				1.5		1.4	0.94	1.49
				2.0		1.7	1.26	1.35
				2.5		2.5	1.58	1.58
				3.0		3.5	1.89	1.85

[a] In contact with remolded soil
[b] Pile restrained
[c] Estimated from standard penetration test
[d] Calculated from field vane tests

one-third the ultimate lateral capacity of a pile) when the unconfined compressive strength of the soil is less than about 1.0 ton per sq ft. However, when the unconfined compressive strength of the soil exceeds about 1.0 ton per sq ft, it is expected that the actual deflections at the ground surface may be considerably larger than the calculated lateral deflections due to the erratic nature of the supporting soil.

However, it should be noted also that only an estimate of the lateral deflections is required for most problems and that the accuracy of the proposed method of analysis is probably sufficient for this purpose. Additional test data are required before the accuracy and the limitations of the proposed method can be established.

ULTIMATE LATERAL RESISTANCE

General.—At low load levels, the deflections of a laterally loaded pile or pole increase approximately linearly with the applied load. As the ultimate capacity is approached, the lateral deflections increase very rapidly with increasing applied load. Failure of free or fixed-headed piles may take place by any of the failure mechanisms shown in Figs. 1 and 2. These failure modes are discussed below.

Unrestrained Piles.—The failure mechanism and the resulting distribution of lateral earth pressures along a laterally loaded free-headed pile driven into a cohesive soil is shown in Fig. 7. The soil located in front of the loaded pile close to the ground surface moves upwards in the direction of least resistance, while the soil located at some depth below the ground surface moves in a lateral direction from the front to the back side of the pile. Furthermore, it has been observed that the soil separates from the pile on its back side down to a certain depth below the ground surface.

J. Brinch-Hansen[47] has shown that the ultimate soil reaction against a laterally loaded pile driven into a cohesive material (based on the assumption that the shape of a circular section can be approximated by that of a square) varies between $8.3c_u$ and $11.4c_u$, where the cohesive strength c_u is equal to half the unconfined compressive strength of the soil. On the other hand, L. C. Reese,[48] M. ASCE has indicated that the ultimate soil reaction increases at failure from approximately 2 c_u at the ground surface to $12c_u$ at a depth of approximately three pile diameters below the ground surface. T. R. McKenzie[49] has found from experiments that the maximum lateral resistance is equal to approximately 8 c_u, while A. G. Dastidar[50] used a value of 8.5 c_u when calculating the restraining effects of piles driven into a cohesive soil. The ultimate lateral resistance has been calculated in Appendix III as a function of the shape at the cross-sectional area and the roughness of the pile

[47] Brinch-Hansen, J., "The Stabilizing Effect of Piles in Clay," C. N. Post, November, 1948. (Published by Christiani & Nielsen, Copenhagen, Denmark).

[48] Reese, L. C., discussion of "Soil Modulus for Laterally Loaded Piles," by B. McClelland and J. A. Focht, Jr., Transactions, ASCE, Vol. 123, 1958, pp. 1071-1074.

[49] McKenzie, T. R., "Strength of Deadman Anchors in Clay," thesis presented to Princeton University, at Princeton, N. J., in 1955, in partial fulfilment of the requirements for the degree of Master of Science.

[50] Dastidar, A. G., "Pilot Tests to Determine the Effect of Piles in Restraining Shear Failure in Clay," Princeton Univ., Princeton, N. J., 1956 (unpublished).

surface. The calculated ultimate lateral resistances varied between 8.28 c_u and 12.56 c_u as can be seen from Table 5.

Repetitive loads, such as those caused by wave forces, cause a gradual decrease of the shear strength of the soil located in the immediate vicinity of the loaded pile. The applied lateral load may cause, in the case where the soil is over-consolidated, a decrease of the pore pressures and as a result, gradual swelling and loss in shear strength may take place as water is absorbed from any available source. Unpublished data collected by the author suggest that repetitive loading could decrease the ultimate lateral resistance of the soil about one-half its initial value. Additional data are however required.

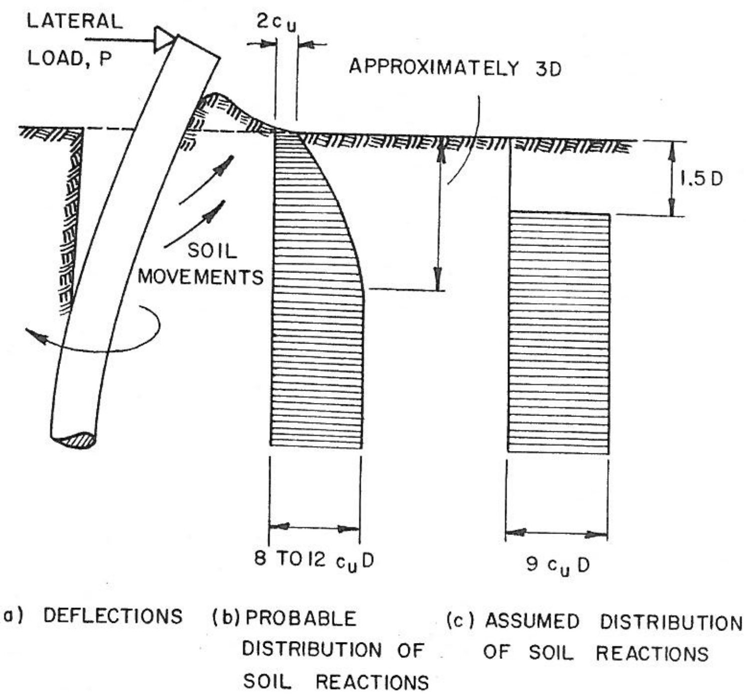


FIG. 7.—DISTRIBUTION OF LATERAL EARTH PRESSURES

The ultimate lateral resistance of a pile group may be considerably less than the ultimate lateral resistance calculated as the sum of the ultimate resistances of the individual piles. N. C. Donovan,[51] A. M. ASCE found no reduction in lateral resistance when the pile spacing exceeded four pile diameters. When the piles were closer than approximately two pile diameters, the piles and the soil located within the pile group behaved as a unit.

[51] Donovan, N. C., "Analysis of Pile Groups," thesis presented to Ohio State University, at Columbus, Ohio, in 1959, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

The probable distribution of lateral soil reactions is shown in Fig. 7(b). On the basis of the measured and calculated lateral resistances, the probable distribution has been approximated by the rectangular distribution shown in Fig. 7(c). It has been assumed that the lateral soil reaction is equal to zero to a depth of 1-1/2 pile diameters and equal to 9.0 $c_u D$ below this depth. The resulting calculated maximum bending moment and required penetration depth (assuming the rectangular distribution of lateral earth pressures shown in Fig. 7c) will be somewhat larger than that corresponding to the probable pressure distribution at failure. Thus the assumed pressure distribution will yield results which are on the safe side.

Short Piles.—The distribution of soil reactions and bending moments along a relatively short pile at failure is shown in Fig. 8. Failure takes place when the soil yields along the total length of the pile, and the pile rotates as a unit

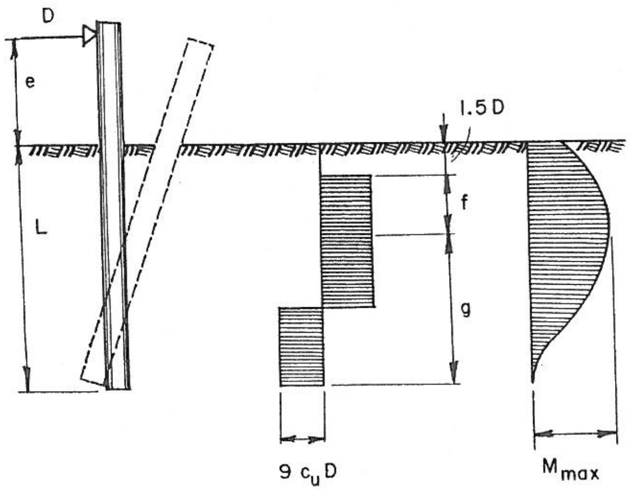


FIG. 8.—DEFLECTION, SOIL REACTION AND BENDING MOMENT DISTRIBUTION FOR A SHORT FREE-HEADED PILE

around a point located at some depth below the ground surface. The maximum moment occurs at the level where the total shear force in the pile is equal to zero at a depth (f + 1.5 D) below the ground surface. The distance f and the maximum bending moment M_{max}^{pos} can then be calculated from the two equations:

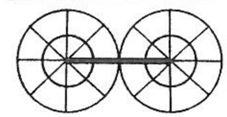
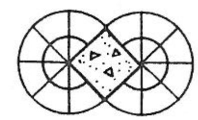
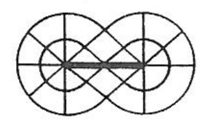
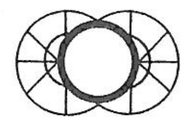
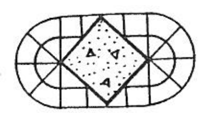
$$f = \frac{P}{9\ c_u D} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

and

$$M_{max}^{pos} = P\ (e + 1.5D + 0.5f) \quad \ldots\ldots\ldots\ldots \quad (13)$$

in which e is the eccentricity of the applied load as defined in Fig. 8. The part of the pile with the length g (located below the point of maximum bending

TABLE 5.—ULTIMATE LATERAL RESISTANCE

SLIP FIELD PATTERN	SURFACE	ULTIMATE LATERAL RESISTANCE, q_{ult}/c_u
	ROUGH	12.56
	ROUGH	11.42
	SMOOTH	11.42
	SMOOTH	9.14
	SMOOTH	8.28

moment) resists the bending moment M_{max}^{pos}. Then from equilibrium requirements

$$M_{max}^{pos} = 2.25\ D\ g^2 \quad \ldots\ldots\ldots\ldots\ldots \quad (14)$$

The ultimate lateral resistance of a short pile driven into a cohesive soil can then finally be calculated from Eqs. 12, 13 and 14 if it is observed that

$$L = (1.5\ D + f + g) \quad \dots\dots\dots\dots\dots \quad (15)$$

The ultimate lateral resistance can also be determined directly from Fig. 9 where the dimensionless ultimate lateral resistance $P_{ult}/c_u D^2$ has been plotted as a function of the dimensionless embedment length L/D. It should be emphasized that it has been assumed in this analysis that failure takes place when the pile rotates as a unit, and that the corresponding maximum bending moment M_{max}^{pos} calculated from Eqs. 13 and 14 is less than the ultimate or yield moment resistance of the pile section M_{yield}.

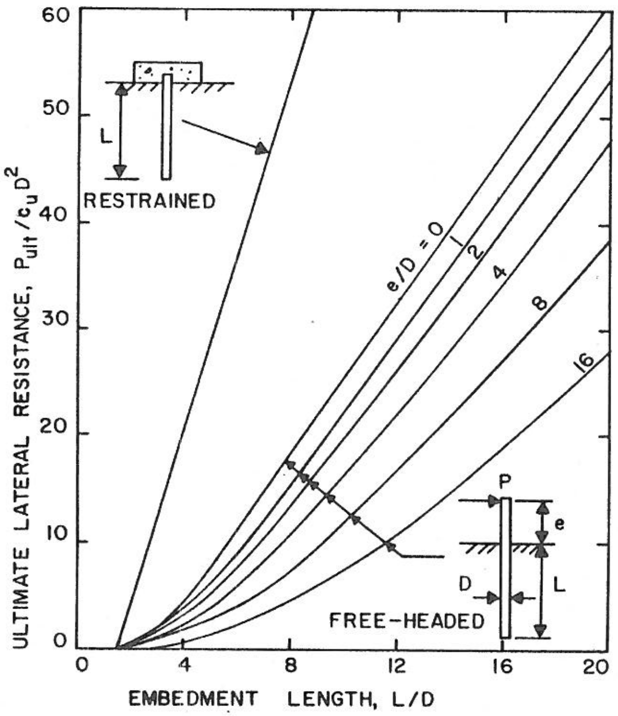


FIG. 9.—COHESIVE SOILS—ULTIMATE LATERAL RESISTANCE

Long Piles.—The mechanism of failure for a long pile when a plastic hinge forms at the location of the maximum bending moment is shown in Fig. 10(a). Failure takes place when the maximum bending moment as calculated from Eq. 13 is equal to the moment resistance of the pile section. The assumed distribution of lateral earth pressures and bending moments is shown in Figs. 10(b) and 10(c). Thus it has been assumed that the lateral deflections are large enough to develop the full passive resistance of the soil down to the depth corresponding to the location of the maximum bending moment in the

pile. The corresponding dimensionless ultimate lateral resistance $P_{ult}/c_u D^2$ has been plotted in Fig. 11 as a function of the dimensionless moment resistance of the pile section $M_{yield}/c_u D^3$ where M_{yield} is the yield strength of the pile section, D the diameter of the pile and c_u, the cohesive strength of the soil.

It can be seen from Fig. 11 that the ultimate lateral resistance increases rapidly with increasing moment resistance of the pile section and with increasing unconfined compressive strength of the soil.

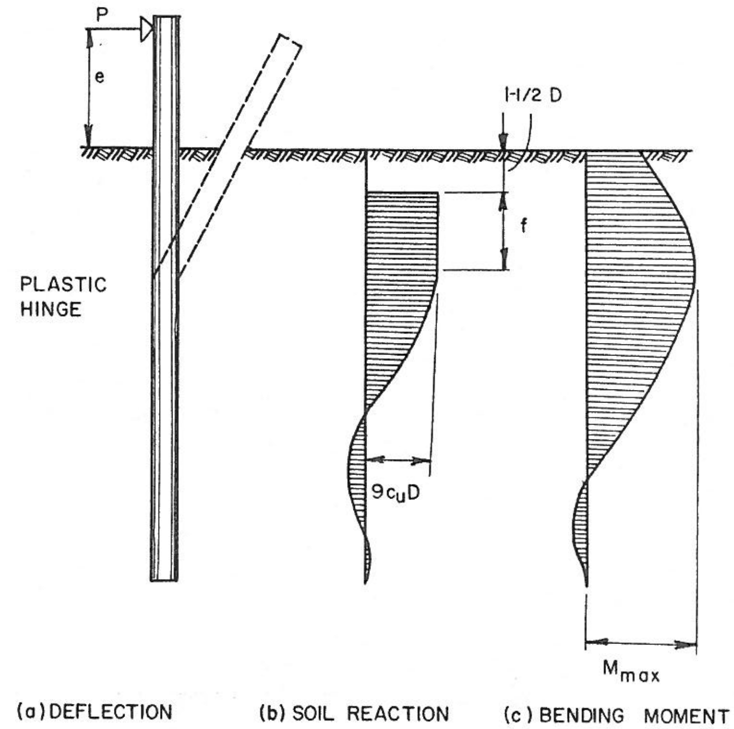


FIG. 10.—DEFLECTION, SOIL REACTION AND BENDING MOMENT DISTRIBUTION FOR A LONG FREE-HEADED PILE

Restrained Piles.

Short Piles.—The mode of failure for a very short restrained pile is shown in Fig. 12. Failure takes place when the applied lateral load is equal to the ultimate lateral resistance of the soil, and the pile moves as a unit through the soil. The corresponding assumed distributions of lateral earth pressures and bending moments are shown in Figs. 12(b) and 12(c), respectively. The ultimate lateral resistance can be calculated from equilibrium requirements as

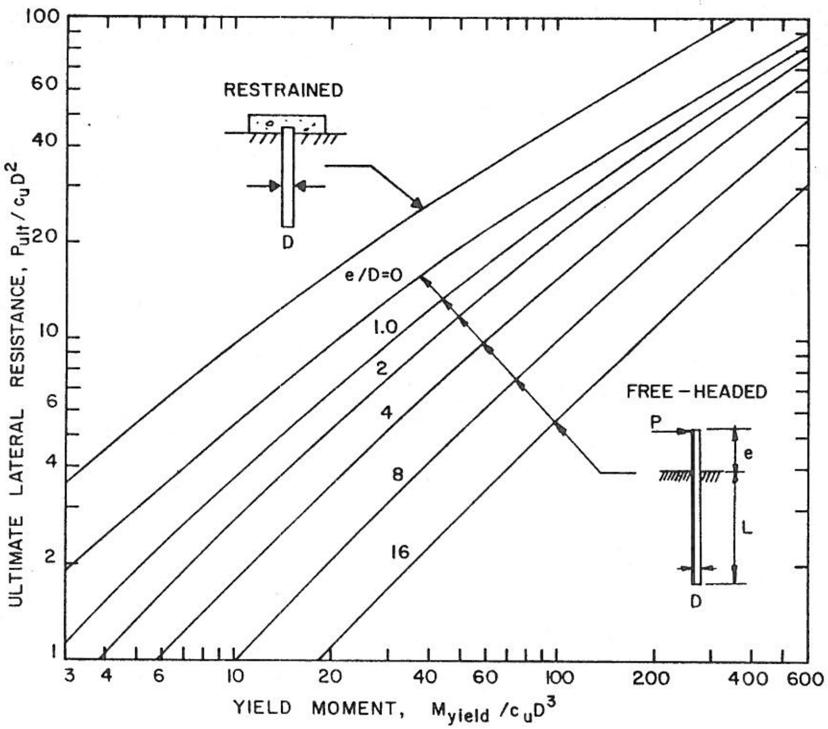


FIG. 11.—COHESIVE SOILS—ULTIMATE LATERAL RESISTANCE

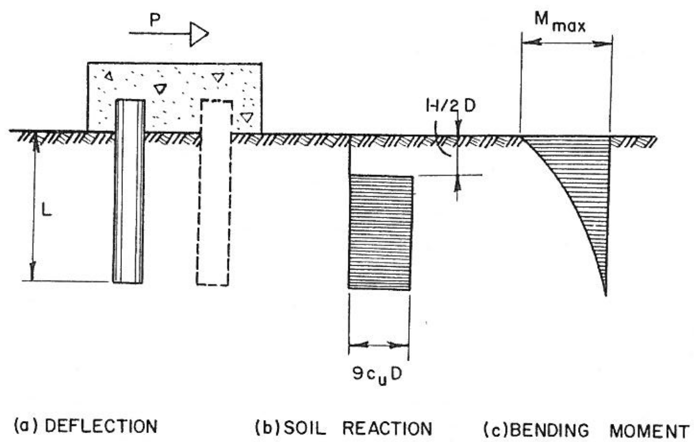


FIG. 12.—DEFLECTION, SOIL REACTION AND BENDING MOMENT DISTRIBUTION FOR A SHORT RESTRAINED PILE

$$P_{ult} = 9\ c_u\ D\ (L - 1.5D) \quad \ldots\ldots\ldots\ldots\ (16)$$

In order for the laterally loaded pile to fail as shown, it is necessary for the maximum negative bending moment M_{max}^{neg} to be less than or equal to the yield moment resistance of the pile section. Thus

$$P_{ult}\ (0.5L + 0.75\ D) \leq M_{yield} \quad \ldots\ldots\ldots\ldots\ (17)$$

Intermediate Length Piles.—The failure mode of intermediate length restrained piles is illustrated in Fig. 13(a). At failure, the restraining moment

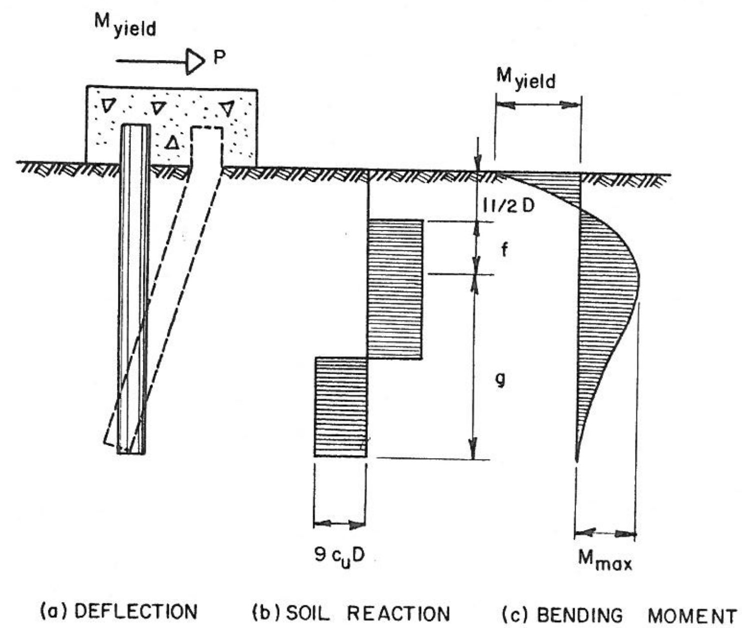
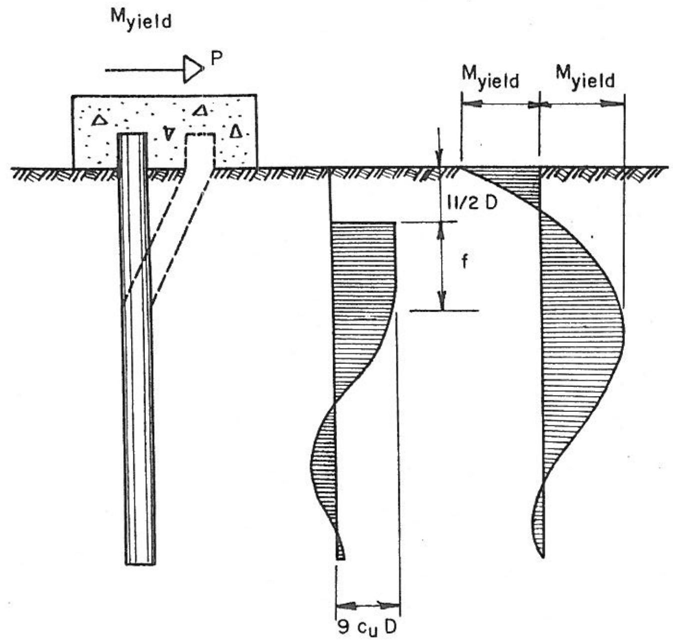


FIG. 13.—DEFLECTION, SOIL REACTION AND BENDING MOMENT DISTRIBUTION FOR A RESTRAINED PILE OF INTERMEDIATE LENGTH

at the head of the pile is, equal to the ultimate moment resistance of the pile section M_{yield}, and the pile rotates around a point located at some depth below the ground surface. The corresponding assumed distribution of lateral earth pressures and bending moments is shown in Figs. 13(b) and 13(c), respectively. The maximum positive bending moment M_{max}^{pos} occurs at a section located at a depth (1.5 D + f) below the ground surface. The depth (1.5 D + f) can be determined from the requirement that the total shear force at the pile section located at this depth must be equal to zero. The ultimate lateral resistance can then be calculated from Eqs. 12 and 13. The corresponding maximum positive bending moment M_{max}^{pos} determined from equilibrium requirements is equal to

$$M_{max}^{pos} = P(1.5\ D + 0.5f) - M_{max}^{neg} \qquad (18)$$

The part of the pile located below the section of maximum positive moment resists the maximum bending moment M_{max}^{pos} calculated from Eq. 14. With a



(a) DEFLECTION (b) SOIL REACTION (c) BENDING MOMENT

FIG. 14.—DEFLECTION, SOIL REACTION AND BENDING MOMENT DISTRIBUTION FOR A LONG RESTRAINED PILE

knowledge of the maximum positive bending moment, the ultimate lateral resistance can finally be calculated from Eqs. 12, 14 and 18. The ultimate lateral resistance can also be determined directly from Fig. 9 in which the dimensionless quantity $P_{ult}/c_u D^2$ has been plotted as a function of the dimensionless length L/D. However, it should be noted that it is necessary for the maximum positive moment M_{max}^{pos} to be less than the yield or ultimate moment resistance of the pile section M_{yield}.

Long Piles.—The mode of failure of a long restrained laterally loaded pile driven into a cohesive soil is shown in Fig. 14. Failure takes place when two plastic hinges form along the pile. The first hinge is located at the section of the maximum negative moment (at the bottom of the pile cap) and the second hinge at the section of maximum positive moment at the depth (1.5 D + f) below the ground surface. The corresponding assumed distributions of the

TABLE 6.—MAXIMUM BENDING MOMENT

Test Pile (1)	Applied Lateral Load P, in kips (2)	Measured Maximum Moment M_{test}, in kip-feet (3)	Calculated Maximum Moment M_{calc}, in kip-feet (4)	Ratio M_{test}/M_{calc} (5)
Parrack,[19] 1952				
1	20	147[a]	137[a,b]	1.07
	40	336	309	1.09
	60	556	497	1.12
	80	800	707	1.13
McCammon and Ascherman,[16] 1953				
1	5.0	385[a]	460[a]	0.84
	10.0	915	930	0.98
	15.0	1375	1405	0.98
2	12.0	400[a]	410[a]	0.98
	24.0	1021	1045	0.98
	36.0	1730	1780	0.97
Osterberg,[18] 1958				
T1	2.91	44.4	47.8	0.93
T2	2.42	38.5	39.7	0.97
T3	1.42	22.5	23.3	0.97
T4	1.42	21.7	23.3	0.93
T5	4.90	74.5	80.7	0.92
T6	3.91	62.0	64.3	0.96
T7	2.91	45.0	47.8	0.94
T14	3.41	52.5	56.2	0.94
			Average	0.945

[a] Sum of maximum positive and negative bending moments
[b] Average unconfined compressive strength taken as 0.500 tons per sq ft

soil reactions and bending moments are shown in Figs. 14(b) and 14(c), respectively. The ultimate lateral resistance can be determined from Eqs. 12 and 18 if it is noted that the maximum bending moment M_{max}^{pos} is equal to the ultimate moment resistance of the pile section M_{yield}. The resulting ultimate lateral resistance is equal to

$$P_{ult} = \frac{2\ M_{yield}}{(1.5D + 0.5\ f)} \quad \ldots\ldots\ldots\ldots\ldots \quad (19)$$

The dimensionless quantity P_{ult}/c_uD^2 is plotted in Fig. 11 as a function of the unconfined compressive strength q_u, the pile diameter D and the yield resistance of the pile section M_{yield}.

Comparison with Test Data.—The maximum bending moments calculated by Eqs. 13 and 18 have been compared in Table 6 with those determined experimentally by Parrack,[19] by McCammon and Ascherman[16] and by Osterberg.[18]

The test results presented by Parrack[19] and by McCammon and Ascherman[16] were obtained from load test on partially restrained piles. The degree of end restraint for the test piles is not known. The degree of end restraint affects the numerical values of both the positive and the negative maximum bending moments. However, it should be noted that the sum of the positive and negative maximum bending moments is independent of the degree of end restraint and that this sum can be determined directly from the equation

$$M_{max}^{pos} + M_{max}^{neg} = P\ (1.5\ D + 0.5\ f) \quad \ldots\ldots\ldots \quad (20)$$

The sum $\left(M_{max}^{pos} + M_{max}^{neg}\right)$ depends only on the dimensions of the test pile, on the applied lateral load and on the cohesive strength of the supporting soil.

For the investigations carried out by Parrack[19] and by McCammon and Ascherman,[16] the sum of the positive and negative bending moment has been compared with the measured value in Table 6. For the investigation carried out by Osterberg,[15] the maximum bending moments have been calculated from Eq. 13. The test data are examined in detail in Appendix II.

It can be seen from Table 6 that the agreement between measured and calculated bending moments is good. The main reason for this very satisfactory agreement is the fact that the calculated maximum bending moment is not very sensitive to small variations in the assumed distribution of lateral earth pressures or to small variations in the measured cohesive strength of the soil. The test data indicate that the proposed method of analysis can be used with confidence to predict the maximum bending moments for both unrestrained and restrained piles. However, it should be noted that the experimental verification is limited. Additional test data are desirable.

SUMMARY AND CONCLUSIONS

Methods for the calculation of the lateral deflections at working loads and the ultimate lateral resistance of laterally loaded piles driven into cohesive saturated soils have been presented. The lateral deflections at working loads have been calculated utilizing the concept of a coefficient of subgrade reaction. Methods for the evaluation of the coefficient of subgrade reaction have been considered and the results have been presented in the form of graphs and tables. The lateral deflections at the ground surface has been expressed

in terms of a dimensionless deflection, a dimensionless length, and the degree of end restraint of the loaded pile.

The ultimate lateral resistance of laterally loaded piles has been calculated assuming that the piles are transformed into a mechanism through the formation of plastic hinges. The ultimate lateral resistance of relatively short laterally loaded piles has been assumed to be governed by the ultimate lateral resistance of the surrounding cohesive soil whereas the ultimate lateral resistance of relatively long piles has been assumed to be governed by the plastic or yield resistance of the pile sections. The ultimate lateral resistances of free and restrained laterally loaded piles has been presented in graphical form.

The calculated deflection at working loads and the calculated maximum bending moments have been compared with available test data. Good agreement was found between calculated and measured values. The available test data are, however, limited and the proposed methods should be applied with caution.

APPENDIX I.—NOTATION

The following notations have been adopted for use in this paper:

B = diameter or width of load plate, in inches;

c_u = cohesion determined from undrained triaxial, direct shear or vane tests, in pounds per square inch;

D = diameter or width of test pile, in inches;

d_o = deflection of load plate, in inches;

E_p = modulus of elasticity of pile material, in pounds per square inch;

E_{50} = secant modulus corresponding to half the ultimate unconfined compressive strength of the soil, in pounds per square inch;

f = distance from 1.5 pile diameters below ground surface to location of maximum bending moment (Fig. 8), in inches;

g = distance from location of maximum bending moment to bottom of pile (Fig. 8), in inches;

I_p = moment of inertia of pile section, in in.^{4};

K = k D, in pounds per square inch;

K_o = k_oB, in pounds per square inch;

K_∞ = $k_\infty D$, in pounds per square inch;

k = coefficient of subgrade reaction, in pounds per cubic inch;

k_m = coefficient of lateral subgrade reaction with respect to moment acting at mid-height of short laterally loaded piles (Fig. 5b), in pounds per cubic inch;

k_o = coefficient of subgrade reaction of square or circular plates, in pounds per cubic inch;

k_p = coefficient of lateral subgrade reaction with respect to load acting at mid-height of short laterally loaded piles (Fig. 5a), in pounds per cubic inch;

k_∞ = coefficient of lateral subgrade reaction for a long laterally loaded pile, in pounds per cubic inch;

L = length of embedment (Fig. 5), in inches;

L' = equivalent length of embedment (Fig. 5), in inches;

n_1 = coefficient (Table 1);

n_2 = coefficient (Table 2);

M = moment, in pounds per inch;

M^{pos}_{max} = maximum positive bending moment, in pounds per inch;

M^{neg}_{max} = maximum negative bending moment, in pounds per inch;

M_{yield} = yield or ultimate moment resistance of pile section, in pounds per inch;

P = lateral load, in pounds;

P_{ult} = ultimate lateral resistance, in pounds;

Q = soil reaction per unit length of pile, in pounds per inch;

q = unit soil reaction, in pounds per square inch;

q_u = unconfined compressive strength, in pounds per square inch;

q_{ult} = ultimate lateral resistance, in pounds per square inch;

y = lateral deflection, in inches;

y_o = lateral deflection at ground surface, in inches;

y_m = lateral deflection at ground surface caused by moment acting at mid-height of short piles (Fig. 5b), in inches;

y_p = deflection at ground surface caused by load acting at mid-height of short piles (Fig. 5a), in inches;

y_{calc} = calculated lateral deflection at ground surface, in inches;

y_{test} = measured lateral deflection at ground surface, in inches;

α = coefficient equal to $n_1 n_2$;

βL = dimensionless length; and

μ = poisson's ratio.

APPENDIX II.—SUMMARY OF TEST DATA

A summary is presented in this Appendix of the soil conditions, the properties of the test piles and of the assumptions made in the analysis of the test data reported in Tables 4 and 6. Each investigation is considered separately.

Shilts, Graves and Driscoll,[24] *1948.*—The deflections of laterally loaded poles placed in augered holes and backfilled with compacted material were measured by Shilts, Graves and Driscoll. The soil at the test site consisted of a thin layer of top soil underlain by a silty clay and a clayey silt with an average unconfined compressive strength of 1.53 tons per sq ft. The lateral deflections of the test poles at the ground surface increased approximately linearly with the applied load. The lateral deflections of the test poles have been calculated assuming that the (E_{50}/q_u)-ratio of the soil is equal to 50 and that the poles were rigid. Pole 9 was set in a precast concrete pipe with an outside diameter of 2 ft and the space between the pole and the concrete pile was filled with compacted soil. The lateral deflection of pole 9 has been calculated assuming that the pole and the concrete pipe move as a unit. The discrepancy between measured and calculated lateral deflections (Table 4) for pole 9 can probably be attributed to movement of the pole with respect to the concrete pipe.

Parrack,[19] *1952.*—Parrack measured the lateral deflections and the bending moment distribution for a partially restrained instrumented pipe pile with an outside diameter of 24.0 in. The test pile was driven through a soft organic saturated silty clay. The average cohesive strength c_u (evaluated as half the unconfined compressive strength) was 370 lb per sq ft, within the significant depth,[52] while the average cohesive strength c_u measured by field vane tests was 500 lb per sq ft.[53] The calculated dimensionless length βL of the test pile was larger than 2.5 ($\beta L = 10.4$). Therefore, it has been assumed in the calculations that the test pile behaved as an infinitely long pile. The lateral deflections have been calculated assuming that the (E_{50}/q_u)-ratio of the silty clay is equal to 50, that the cohesive strength as measured by the field vane test is representative of the average cohesive strength of the soil in place and that the test pile was fully free at the ground surface. (The inflection point of the pile was located approximately at the ground surface.)

The test pile was subjected to several load cycles at each load level. The deflection y_{test} is the measured lateral deflection at the ground surface during the first load cycle before any appreciable consolidation and creep have

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52 "Foundation Investigation, Bay Marchand "E" and "F" Structures," Greer & McClelland, Consulting Engrs., Houston, Texas, December, 1949.

53 "Supplementary Foundation Investigation, Bay Marchand "E" Structure, Lafourch Parish, Louisiana," Greer & McClelland, Consulting Engrs., Houston, Texas, February, 1954.

taken place. The lateral deflections at working loads were found to be approximately equal to the calculated lateral deflection corresponding to the fully free condition as can be seen from Table 4. The measured lateral deflections increased rapidly as the maximum applied load was approached and exceeded at the applied lateral load of 45 kips the calculated lateral deflections.

The maximum and negative bending moments in a laterally loaded restrained pile depends on the degree of end restraint which is unknown for the test pile. It should be noted, however, that the sum of the maximum positive and the maximum negative bending moments is independent of the degree of end restraint and can be calculated directly from Eq. 20. The shearing strength of the soil c_u within the significant depth has been taken as 500 lb per sq ft. The calculated maximum bending moments are relatively insensitive to variations in the assumed cohesive strength. If a value of 370 lb per sq ft is taken as the cohesive strength, then the calculated bending moments will be from 4% to 10% larger than those corresponding to a cohesive strength of 500 lb per sq ft.

McCammon and Ascherman,[16] *1953.*—The distribution of lateral bending moment in a free-headed and a restrained laterally loaded pile with a diameter of 4.5 ft was measured by McCammon and Ascherman.[16] The test piles were driven into the bottom of Lake Maracaibo, Venezuela and the soil within the significant depth consisted of a very soft gray clay with an average water content of 150%, an average liquid limit of 85% and an average plasticity index of 45%. No information is available concerning the deformation and strength properties of the soil. Since the shallow bottom sediments probably are normally consolidated, it is reasonable to expect that the shearing strength of the soil will increase linearly with depth corresponding to a $c_u/\bar{p}$ ratio of 0.27.[54] The calculated average submerged unit weight of the soil is 21 lb per cu ft, assuming a unit specific weight of 2.7 for the solids and an average water content of 150%. The maximum lateral bending moment for the unrestrained pile denoted (1) has been calculated from Eq. 13. For the restrained pile denoted (2), the sum of the maximum positive and negative bending moments has been calculated from Eq. 20. The calculated and the measured maximum bending moments have been compared in Table 6. Good agreement was found between measured and calculated values.

The calculated maximum bending moments are insensitive to variations in the assumed cohesive strength of the clay. If it is assumed that the $c_u/\bar{p}$-ratio is twice the value assumed previously, then the calculated bending moments will decrease by about 2% and the ratio M_{test}/M_{calc} will increase by the same amount.

McNulty,[17] *1956.*—The lateral resistance of two pile groups consisting each of three vertical timber piles with a diameter of 12 in. was measured. The piles were restrained by a pile cap. The test piles were driven through 9 ft of clay with an average standard penetration resistance of 6 blows per ft. The calculated significant depth is approximately equal to 10 ft. The lateral resistance is governed by the average shear strength of the shallow clay stratum. The lateral deflections of the test piles has been calculated from Eq. 5(b) assuming that the piles were fully restrained and that the modulus of

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54 Bjerrum, L., and Simons, N. E., "Comparison of Shear Strength Characteristics of Normally Consolidated Clays," Research Conf. on Shear Strength of Cohesive Soils, Boulder, Colo., June, 1960, pp. 711-726.

elasticity of the pile material is 1.5×10^6 psi. The coefficient of subgrade reaction has been calculated from Eq. 6 assuming that the unconfined compressive strength of the soil (in tons per square foot) is 1/5 the standard penetration resistance N (in blows per foot). Since the average measured standard penetration resistance was 6 blows per ft, the unconfined compressive strength q_u of the soil has been taken as 1.2 tons per sq ft. The calculated lateral deflections have been compared with the measured deflections in Table 4. The calculated lateral deflections compares well with the measured values at half the maximum applied lateral load of 20 kips.

The calculated lateral deflections are insensitive to the assumed value of the coefficient of subgrade reaction. If the unconfined compressive strength (in tons per square foot) is taken as 1/8 the standard penetration resistance N (in blows per foot), then the lateral deflections calculated in Table 4 are increased by 13%.

Osterberg,[18] *1958.*—Osterberg carried out an extensive series of tests on laterally loaded poles placed in holes predrilled in a stiff to very stiff clay. The diameter of the predrilled holes was approximately 2 in. larger than the diameter of the corresponding test pile. The space between the test piles and the sides of the predrilled holes was filled with sand which was compacted in layers or vibrated by striking the side of the pile by a hammer. The dimensionless lengths of the test piles βL were less than 1.5. The deflections at the ground surface have been calculated assuming that the test piles were infinitely stiff. The tests were not carried out to failure and the ultimate lateral resistances of the piles are therefore not known. The deflections at the ground surface were calculated at working loads (one-third the maximum applied lateral load).

The unconfined compressive strength used in the calculations of the lateral deflections is the average strength measured for samples obtained from the bore hole located closest to the test pile being considered. Stress-strain relationships obtained from unconfined compression tests indicate that the average (E_{50}/c_u)-ratio of the soil was approximately 50. The agreement between measured and calculated lateral deflections is considered satisfactory. The average value of the ratio y_{test}/y_{calc} is 1.65. The deviation between the measured and the calculated values may be partly attributed to compression of the sand placed in the space between the test piles and the sides of the predrilled auger holes.

A comparison between measured and calculated bending moments at the maximum applied lateral load is made in Table 6. The maximum bending moments have been calculated from Eq. 13, assuming that the cohesive strength of the soil is equal to half the average unconfined compressive strength measured for samples obtained from the bore hole located closest to the respective test pile. Good agreement was found between measured and calculated maximum bending moments.

Peck and Davisson,[21] *1962.*—The lateral deflections of a steel H-pile driven to bedrock through approximately 30 ft of black to gray, organic silt were measured by means of a slope indicator. The shearing strength c_u of the organic silt, measured by field vane tests, increased from zero at the mudline to approximately 400 lb per sq ft at a depth of 27 ft below the mudline. The lateral deflections have been calculated by Eqs. 5a, 6, 7 and 11, at the mudline assuming that the average shearing strength c_u is 200 lb per sq ft and that the secant modulus of the soil is 100 c_u. The calculated lateral deflections compare favorably with the measured values as shown in Table 4.

APPENDIX III

The lateral resistance which develops along a laterally loaded pile moving through a cohesive soil can be calculated approximately by means of the theory of plasticity. The ultimate lateral resistance can be calculated assuming that slip or rupture of the soil takes place along plane or spiral shaped failure surfaces.[22,55,56]

At failure, the soil located close to the ground surface to a depth of approximately three pile diameters moves in an upwards direction towards the ground surface while the soil located below this depth, moves in the lateral direction from the front to the back side of the pile. The ultimate lateral resistance can be calculated, below a depth of approximately three pile diameters, assuming plain strain conditions and that the soil is weightless.

Slip or rupture of a soil with a cohesion c and friction angle ϕ can be assumed to take place along two families of failure or rupture surfaces which are inclined at an angle of $(90° + \phi)$ degrees with each other. When the soil behaves as a weightless material the principal stresses are constant within the area where the two families of slip surfaces are plane.

The variation of the major principal stresses along a spiral shaped failure surface can be calculated from the equation:

$$\left(p_1\right)_a = \left(p_1\right)_b e^{-2v\tan\phi} + c\left(e^{-2v\tan\phi} - 1\right)/\tan\phi \quad \ldots\ldots (21)$$

in which $(p1)_a$ is the major principal stress at point a, $(p1)_b$ the major principal stress at point b, and v the change of inclination (slope) of the failure surface when traveling from point a to point b. For a frictionless ($\phi = 0$) material, Eq. 21 can be simplified to

$$\left(p_1\right)_a = \left(p_1\right)_b - 2c_u v \quad \ldots\ldots\ldots\ldots\ldots (22)$$

in which c_u is the cohesive strength of the frictionless material.

The ultimate lateral resistances of various pile sections with smooth or rough surfaces have been calculated in Table 5. The calculated lateral unit resistance varied between 8.28 c_u and 12.56 c_u. It can be seen from Table 5 that the calculated ultimate lateral resistance does not vary between wide limits. The calculated ultimate lateral resistances are not affected appreciably by the roughness or by the shape at the pile section. A value of 9 c_u has therefore been used in the calculations of the ultimate lateral unit resistance of a cohesive soil.

[55] "Plasticity in Engineering," by F. K. T. Iterson, Hafner Publishing Co., Inc., New York, 1947.

[56] "Earth Pressure Calculation," by J. Brinch-Hansen, The Danish Technical Press, Copenhagen, 1953.